

$$\begin{cases} x(0) = 1 \\ y(0) = 2 \\ z(0) = 3 \end{cases} \quad \text{בהתקיים:} \quad \begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases}$$

עליך לפתור את מערכת המשוואות

פתרון:

$$\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases} \Rightarrow z = \dot{x} - y \Rightarrow \begin{cases} \dot{x} - \dot{y} = y - x \\ \dot{x} - \dot{z} = z - x \\ \dot{y} - \dot{z} = z - y \end{cases} \Rightarrow \begin{cases} x + \dot{x} = y + \dot{y} \\ x + \dot{x} = z + \dot{z} \\ y + \dot{y} = z + \dot{z} \end{cases} \Rightarrow$$

$$x + \dot{x} = y + \dot{y} = z + \dot{z} \Rightarrow x(t), y(t), z(t) \text{ are all of same general form}$$

$$\begin{cases} \dot{x} = y + z \\ \dot{y} = x + z \\ \dot{z} = x + y \end{cases} \Rightarrow \begin{cases} \dot{z} = \ddot{x} - \dot{y} \\ \dot{z} = \ddot{y} - \dot{x} \\ \dot{z} = x + y \end{cases} \Rightarrow \begin{cases} \ddot{x} - \dot{y} = \ddot{y} - \dot{x} \\ \ddot{x} - \dot{y} = x + y \\ \ddot{y} - \dot{x} = x + y \end{cases} \Rightarrow$$

$$\begin{cases} \dot{x} - \dot{y} = y - x \\ \ddot{x} - \dot{y} = x + y \end{cases} \Rightarrow \ddot{x} - \dot{x} = 2x$$

$$\begin{cases} \ddot{x} - \dot{x} - 2x = 0 \\ \ddot{y} - \dot{y} - 2y = 0 \\ \ddot{z} - \dot{z} - 2z = 0 \end{cases} \Rightarrow \begin{cases} \lambda^2 - \lambda - 2 = 0 \\ \lambda^2 - \lambda - 2 = 0 \\ \lambda^2 - \lambda - 2 = 0 \end{cases} \Rightarrow \lambda_1 = -1, \lambda_2 = 2$$

$$Q(t) = 0 \Rightarrow \text{Homogeneous}$$

$$\lambda_1 \neq \lambda_2 \Rightarrow f_h = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \Rightarrow f = f_h = C_1 e^{-t} + C_2 e^{2t}$$

$$\begin{cases} x = C_1 e^{-t} + C_2 e^{2t} \\ y = C_3 e^{-t} + C_4 e^{2t} \\ z = C_5 e^{-t} + C_6 e^{2t} \end{cases} \begin{cases} x(0) = 1 \\ y(0) = 2 \\ z(0) = 3 \end{cases} \Rightarrow \begin{cases} 1 = C_1 + C_2 \\ 2 = C_3 + C_4 \\ 3 = C_5 + C_6 \end{cases}$$

$$\begin{cases} x = C_1 e^{-t} + C_2 e^{2t} \\ y = C_3 e^{-t} + C_4 e^{2t} \\ z = C_5 e^{-t} + C_6 e^{2t} \end{cases} \Rightarrow \begin{cases} \dot{x} = -C_1 e^{-t} + 2C_2 e^{2t} \\ \dot{y} = -C_3 e^{-t} + 2C_4 e^{2t} \\ \dot{z} = -C_5 e^{-t} + 2C_6 e^{2t} \end{cases}$$

$$x + \dot{x} = y + \dot{y} = z + \dot{z} \Rightarrow 3C_2 e^{2t} = 3C_4 e^{2t} = 3C_6 e^{2t} \Rightarrow C_2 = C_4 = C_6$$

$$\begin{cases} x = C_1 e^{-t} + C_2 e^{2t} \\ y = C_3 e^{-t} + C_2 e^{2t} \\ z = C_5 e^{-t} + C_2 e^{2t} \end{cases} \Rightarrow \begin{cases} \dot{x} = -C_1 e^{-t} + 2C_2 e^{2t} \\ \dot{y} = -C_3 e^{-t} + 2C_2 e^{2t} \\ \dot{z} = -C_5 e^{-t} + 2C_2 e^{2t} \end{cases}$$

$$z = \dot{x} - y \quad \Rightarrow \quad C_5 e^{-t} + C_2 e^{2t} = -(C_1 + C_3) e^{-t} + C_2 e^{2t} \quad \Rightarrow \quad C_5 = -(C_1 + C_3)$$

$$\begin{cases} 1 = C_1 + C_2 \\ 2 = C_3 + C_2 \\ 3 = -C_1 - C_3 + C_2 \end{cases} \Rightarrow \begin{cases} 1 = -C_1 + C_3 \\ 1 = -C_1 - 2C_3 \end{cases} \Rightarrow C_3 = 0$$

$$\begin{cases} -1 = C_1 \\ 2 = C_2 \\ 0 = C_3 \end{cases} \Rightarrow C_5 = 1$$

$$\begin{cases} x = -e^{-t} + 2e^{2t} \\ y = 0e^{-t} + 2e^{2t} \\ z = 1e^{-t} + 2e^{2t} \end{cases}$$