$$(1+x^2)y'' + 2xy' + 3x^{-2} = 0$$ (3)

$$(1+x^2)v' + 2xv = -3x^{-2} \quad => \quad v' + \frac{2x}{1+x^2}v = -\frac{3}{x^2(1+x^2)}$$

$y' + P(x)y = Q(x)$

נוסחה: $y = \left[C + \int Q(x)\, e^{\int P(x)dx}\, dx\right] e^{-\int P(x)dx}$

$$P_{(x)} = \frac{2x}{1+x^2} \quad , \quad Q_{(x)} = -\frac{3}{x^2(1+x^2)}$$

$$v_{(x)} = \left[C - 3\int \frac{1}{x^2(1+x^2)} \cdot e^{\int \frac{2x}{1+x^2}dx}\, dx\right] e^{-\int \frac{2x}{1+x^2}dx}$$

$$v_{(x)} = \left[C - 3\int \frac{1}{x^2(1+x^2)} \cdot e^{\ln(1+x^2)}\, dx\right] e^{-\ln(1+x^2)}$$

$$v_{(x)} = \left[C - 3\int \frac{1}{x^2(1+x^2)}(1+x^2)\, dx\right](1+x^2)^{-1}$$

$$v_{(x)} = \left[C - 3\int \frac{1}{x^2}\, dx\right](1+x^2)^{-1}$$

$$v_{(x)} = \left[C + \frac{3}{x}\right]\frac{1}{1+x^2}$$

$$v_{(x)} = \frac{C}{1+x^2} + \frac{3}{x(1+x^2)}$$

$$y_{(x)} = C\int \frac{1}{1+x^2}\,dx + 3\int \frac{1}{x(1+x^2)}\,dx$$

$$\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+D}{1+x^2} = \frac{(A+B)x^2 + DX + A}{x(1+x^2)} \quad => \quad \begin{cases} B = -A \\ D = 0 \\ A = 1 \end{cases}$$

$$\frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}$$

$$y_{(x)} = C\int \frac{1}{1+x^2}\,dx + 3\int \frac{1}{x}\,dx - \frac{3}{2}\int \frac{2x}{1+x^2}\,dx$$

$$y_{(x)} = C \cdot arctgx + 3\ln|x| - \frac{3}{2}\ln(1+x^2) + D$$