

$$. (y' = v \text{ oder } 1547) \quad y'' + x(y')^2 = 0 \quad (1)$$

$$v' + xv^2 = 0 \quad \Rightarrow \quad \frac{dv}{dx} + xv^2 = 0 \quad \Rightarrow \quad \frac{dv}{v^2} + xdx = 0$$

$$-\int \frac{dv}{v^2} - \int xdx = 0 \quad \Rightarrow \quad \frac{1}{v} - \frac{x^2}{2} = C \quad \Rightarrow \quad \frac{dx}{dy} = \frac{x^2}{2} + C \quad \Rightarrow$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x^2 + C}{2} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2}{x^2 + C} \quad \Rightarrow \quad dy = \frac{2}{x^2 + C} dx \quad \Rightarrow$$

$$\Rightarrow \quad dy = 2 \frac{1}{(x+C)(x-C)} dx$$

$$\frac{1}{(x+C)(x-C)} = \frac{A}{x+C} + \frac{B}{x-C} = \frac{(A+B)x + (B-A)C}{(x+C)(x-C)} \quad \Rightarrow \quad B = -A \quad \Rightarrow \quad -2AC = 1 \quad \Rightarrow$$

$$\Rightarrow \quad A = -\frac{1}{2C} \quad \Rightarrow \quad B = \frac{1}{2C} \quad \Rightarrow \quad \frac{1}{(x+C)(x-C)} = \frac{-\frac{1}{2C}}{x+C} + \frac{\frac{1}{2C}}{x-C} = \frac{1}{2C} \left(\frac{1}{x-C} - \frac{1}{x+C} \right)$$

$$dy = 2 \frac{1}{2C} \left(\frac{1}{x-C} - \frac{1}{x+C} \right) dx \quad \Rightarrow \quad dy = \frac{1}{C} \left(\frac{1}{x-C} - \frac{1}{x+C} \right) dx \quad \Rightarrow$$

$$\Rightarrow \quad y = \frac{1}{C} \int \left(\frac{1}{x-C} - \frac{1}{x+C} \right) dx \quad \Rightarrow \quad y = \frac{1}{C} [\ln(x-C) - \ln(x+C)] + D$$

$$y = \frac{1}{C} \ln \frac{x-C}{x+C} + D$$