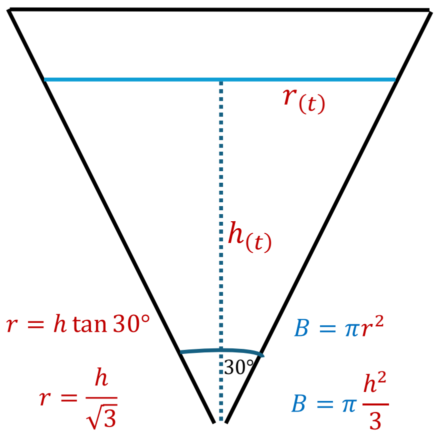
6) חוק טוריצ'לי (זרימת מים דרך חריר): במיכל קוני בעל חתך מעגלי וזווית ראש של 60°, יש חריר בעל שטח של 0.5 סמ"ר בקודקוד. המיכל מלא מים ובזמן $t=0$ פותחים את החריר והמים זורמים החוצה. גובה המים ההתחלתי: מטר $h(0) = 1$.
מהו הזמן הנדרש למיכל להתרוקן?

$$\frac{dh}{dt} = -C_d \cdot A \cdot \sqrt{2g}\,\frac{h^{1/2}}{B(h)}$$

- $\frac{dh}{dt}$: The rate of change of the water height (h) with respect to time (t). The negative sign indicates that the water level is decreasing.
- 0.6: The **discharge coefficient** (C_d). Because of friction and the narrowing of the fluid stream as it exits (called the *vena contracta* effect), the actual flow rate is slightly less than the theoretical maximum. The value of 0.6 is standard for a sharp-edged circular orifice.
- A: The cross-sectional area of the drain hole (orifice) at the bottom.
- g: The acceleration due to gravity (approximately 9.81 m/s^{2}or 32.2 ft/s^2).
- $h^{1/2}$**(or $\sqrt{h}$)**: The square root of the water's height.
  According to Torricelli's law, the velocity of the exiting fluid is directly proportional to $\sqrt{2gh}$.
- $B(h)$: The cross-sectional area of the tank itself, written as a function of the height h.
  If the tank is a perfect cylinder, $B(h)$ is a constant; if the tank is a cone, bowl, or another shape, the surface area $B(h)$ changes as the water level drops.



$$\frac{dh}{dt} = -C_d \cdot A \cdot \sqrt{2g}\,\frac{h^{\frac{1}{2}}}{\pi\frac{h^2}{3}} \quad => \quad \frac{dh}{dt} = -\frac{3}{\pi}C_d \cdot A \cdot \sqrt{2g} \cdot \frac{1}{h^{\frac{3}{2}}} \quad =>$$

$$=> \quad h^{\frac{3}{2}}dh + \frac{3}{\pi}C_d \cdot A \cdot \sqrt{2g} \cdot dt = 0$$

$$\int h^{\frac{3}{2}}dh + \frac{3}{\pi}C_d \cdot A \cdot \sqrt{2g}\int dt = C \qquad g = 980\ cm/s$$

$$\frac{2}{5}h_{(t)}^{\frac{5}{2}} + \frac{3}{\pi}C_d \cdot A \cdot \sqrt{2g} \cdot t = C \quad \rightarrow \quad \frac{2}{5}h_{(t)}^{\frac{5}{2}} + \frac{3}{\pi} \cdot 0.6 \cdot 0.5 \cdot \sqrt{2 \cdot 980} \cdot t = C \quad =>$$

$$\frac{2}{5}h_{(t)}^{\frac{5}{2}} + 12.7t = C \quad , \quad h_{(0)} = 100cm \quad => \quad \frac{2}{5} \cdot 100^{\frac{5}{2}} = C = 40{,}000$$

$$\frac{2}{5}h_{(t)}^{\frac{5}{2}} + 12.7t = 40{,}000 \quad => \quad when\ h = 0 \quad we\ get \quad 12.7t = 40{,}000 \quad =>$$

$$=> \quad t = \frac{40{,}000}{12.7} \quad => \quad t = 3150\ sec = 52.5\ min$$