$$y\,dx + \left(2xy - e^{-2y}\right) dy = 0$$

$$\begin{cases} P_{(x,y)} = y & \rightarrow \quad P_y = 1 \\ Q_{(x,y)} = 2xy - e^{-2y} & \rightarrow \quad Q_x = 2y \end{cases} \quad => \quad P_y \neq Q_x \quad => \quad not\ exact$$

גורם אינטגרציה (ρ)

מעביר משוואה דיפ' לא מדויקת למדויקת.

נתון:

$P\,dx + Q\,dy = 0$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

$\downarrow \qquad \downarrow$

$P_y \qquad Q_x$

לא מעגו"ם!

מנחשים $\rho = \rho(x)$! $\qquad \rho = e^{\int F(x)\,dx}, \quad F(x) = \frac{P_y - Q_x}{Q}$

או $\rho = \rho(y)$! $\qquad \rho = e^{\int F(y)\,dy}, \quad F(y) = \frac{Q_x - P_y}{P}$

$P_1 = \rho \cdot P$

$Q_1 = \rho \cdot Q$

$\Longrightarrow$ פותרים $\quad \int_{x_0}^{x} P_1(x,y)\,dx + \int_{y_0}^{y} Q_1(x_0,y)\,dy = 0$

$$F_{(y)} = \frac{Q_x - P_y}{p} = \frac{2y - 1}{y} = 2 - \frac{1}{y} \quad => \quad \rho_{(y)} = e^{\int\left(2-\frac{1}{y}\right)dy} = e^{2y - \ln|y|} = \frac{e^{2y}}{e^{\ln|y|}} = \frac{e^{2y}}{y}$$

$$\begin{cases} P_{1(x,y)} = \rho P_{(x,y)} = e^{2y} & \rightarrow \quad P_{1y} = 2e^{2y} \\ Q_{1(x,y)} = \rho Q_{(x,y)} = \frac{e^{2y}}{y}\left(2xy - e^{-2y}\right) = 2xe^{2y} - \frac{1}{y} & \rightarrow \quad Q_{1x} = 2e^{2y} \end{cases} \quad => \quad P_{1y} = Q_{1x} \quad => \quad exact$$

$$\int_{x_0}^{x} e^{2y}\,dx + \int_{y_0}^{y} \left(2x_0 e^{2y} - \frac{1}{y}\right) dy = 0$$

$$\left[xe^{2y}\right|_{x_0}^{x} + \left[x_0 e^{2y} - \ln|y|\right|_{y_0}^{y} = 0$$

$$xe^{2y} - x_0 e^{2y} + x_0 e^{2y} - \ln|y| - x_0 e^{2y_0} + \ln|y_0|$$

$$xe^{2y} - \ln|y| = C$$