מד"ר ליניארית מסדר 1

$$y' + P_{(x)}y = Q_{(x)}$$

אם $Q_{(x)} = 0$ אז המשוואה היא הומוגנית.

נוסחה לפתרון המשוואה:

$$y_{(x)} = \left[C + \int Q_{(x)} \cdot e^{\int P_{(x)} \cdot dx}\, dx\right] e^{-\int P_{(x)} \cdot dx}$$

פתור את המד"ר הבאה:

$$\begin{cases} y' + \cot(x) \cdot y = \dfrac{2}{\sin(x)} \\ y_{(\pi/2)} = 1 \end{cases}$$

$$P_{(x)} = \cot(x) \quad , \quad Q_{(x)} = \frac{2}{\sin(x)}$$

$$y_{(x)} = \left[C + \int \frac{2}{\sin(x)} \cdot e^{\int \cot(x) dx}\, dx\right] e^{-\int \cot(x) dx}$$

$$\int \cot(x)\, dx = \int \frac{\cos x}{\sin x} dx \quad \begin{cases} u = \sin x \\ du = \cos x\, dx \end{cases} = \int \frac{du}{u} = ln|\sin x|$$

$$y_{(x)} = \left[C + \int \frac{2}{\sin(x)} \cdot e^{ln|\sin x|}\, dx\right] e^{-l\ \ |\sin x|}$$

$$y_{(x)} = \left[C + \int \frac{2}{\sin(x)} \cdot |\sin x|\, dx\right] \frac{1}{|\sin x|}$$

$$for\ 0 < \sin x$$

$$y_{(x)} = \left[C + \int 2\,dx\right]\frac{1}{\sin x} \quad => \quad y_{(x)} = [C + 2x]\frac{1}{\sin x}$$

$$y_{(x)} = \frac{2x + C}{\sin x}$$

$$y_{(\pi/2)} = 1 \quad => \quad 1 = \frac{\pi + C}{\sin(\pi/2)} \quad => \quad C = 1 - \pi$$

$$y_{(x)} = \frac{2x + 1 - \pi}{\sin x}$$

$$for\ \ \sin x < 0$$

$$y_{(x)} = \left[C + \int -2\,dx\right] \cdot \frac{-1}{\sin x} \quad => \quad y_{(x)} = [C - 2x] \cdot \frac{-1}{\sin x}$$

$$y_{(x)} = \frac{2x - C}{\sin x}$$

$$y_{(\pi/2)} = 1 \quad => \quad 1 = \frac{\pi - C}{\sin(\pi/2)} \quad => \quad C = \pi - 1$$

$$y_{(x)} = \frac{2x - \pi + 1}{\sin x}$$

$$Conclusion: for\ any\ \ \sin x \neq 0$$

$$y_{(x)} = \frac{2x + 1 - \pi}{\sin x}$$