

(2) פתור משוואת גרס' גרס' הבלו

$$\frac{xdx}{1+y} - \frac{ydy}{1+x} = 0 \quad (3)$$

$$(1+e^x)yy' - e^x = 0 \quad (4)$$

$$(1+y^2)dx + xydy = 0 \quad (5)$$

$$xy(1+x^2)y' = 1+y^2 \quad (6)$$

$$(1+y^2)dx - xdy = 0 \quad (7)$$

$$א) (1+y^2)dx + xydy = 0 / \begin{cases} \text{כפול 2} \\ \text{חלקי } x \\ \text{חלקי } (1+y^2) \end{cases} \Rightarrow 2 \frac{dx}{x} + \frac{2ydy}{1+y^2} = 0$$

$$2 \int \frac{dx}{x} + \int \frac{2ydy}{1+y^2} = 0 \Rightarrow 2 \ln|x| + \ln(1+y^2) = C \Rightarrow$$

$$\Rightarrow \ln x^2 + \ln(1+y^2) = C \Rightarrow \ln[x^2(1+y^2)] = C \Rightarrow$$

$$\Rightarrow x^2(1+y^2) = C$$

$$ב) xy(1+x^2) \frac{dy}{dx} = 1+y^2 / \begin{cases} \text{כפול } dx \\ \text{חלקי } x(1+x^2) \\ \text{חלקי } (1+y^2) \end{cases} \Rightarrow \frac{ydy}{1+y^2} = \frac{dx}{x(1+x^2)}$$

$$\text{פירוק לשברים חלקיים} \begin{cases} \frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2} = \frac{(A+B)x^2+Cx+A}{x(1+x^2)} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=0 \end{cases} \\ \frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2} \end{cases}$$

$$\frac{ydy}{1+y^2} = \frac{dx}{x} - \frac{xdx}{1+x^2} \Rightarrow \frac{2ydy}{1+y^2} + \frac{2xdx}{1+x^2} - 2 \frac{dx}{x} = 0$$

$$\int \frac{2ydy}{1+y^2} + \int \frac{2xdx}{1+x^2} - 2 \int \frac{dx}{x} = C \Rightarrow \ln(1+y^2) + \ln(1+x^2) - 2 \ln|x| = C$$

$$\ln \frac{(1+y^2)(1+x^2)}{x^2} = C \Rightarrow \frac{(1+y^2)(1+x^2)}{x^2} = C$$

$$\lambda) (1 + y^2)dx - xdy = 0 \quad \Rightarrow \quad \frac{dx}{x} - \frac{dy}{1 + y^2} = 0$$

$$\int \frac{dx}{x} - \int \frac{dy}{1+y^2} = C \quad \Rightarrow \quad \ln|x| - \arctan(y) = C$$

$$\Upsilon) \frac{xdx}{1+y} - \frac{ydy}{1+x} = 0 \quad \Rightarrow \quad x(1+x)dx - y(1+y)dy = 0 \quad \Rightarrow$$

$$\Rightarrow (x^2 + x)dx - (y^2 + y)dy = 0$$

$$\int (x^2 + x)dx - \int (y^2 + y)dy = C$$

$$\frac{x^3}{3} + \frac{x^2}{2} - \frac{y^3}{3} - \frac{y^2}{2} = C \quad \Rightarrow \quad 2x^3 + 3x^2 - 2y^3 - 3y^2 = C \quad \Rightarrow$$

$$2(x^3 - y^3) + 3(x^2 - y^2) = C$$

$$\eta) (1 + e^x)y \frac{dy}{dx} - e^x = 0 \quad \Rightarrow \quad ydy - \frac{e^x dx}{1 + e^x} = 0$$

$$\int ydy - \int \frac{e^x dx}{1+e^x} = C \quad \Rightarrow \quad \frac{y^2}{2} - \ln(1 + e^x) = C$$