1) פתור את המשו' הבאות:

א) $(y^2-3x^2)\,dx+2xy\,dy=0$

ב) $(x+y)\,dx-(x-y)\,dy=0$

$$\begin{cases} P_{(x,y)} = y^2 - 3x^2 & \rightarrow \quad \dfrac{\partial P}{\partial y} = 2y \\ Q_{(x,y)} = 2xy & \rightarrow \quad \dfrac{\partial Q}{\partial x} = 2y \end{cases} \quad => \quad \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \quad => \quad exact$$

<u>מש' דיפרנציאלית מדויקת:</u>

$P(x,y)\,dx + Q(x,y)\,dy = 0$

$\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ תנאי לקיום מדויקת:

הפתרון: $\displaystyle\int_{x_0}^{x} P(x,y)\,dx + \int_{y_0}^{y} Q(x_0,y)\,dy = 0$

$$\int_{x_0}^{x} (y^2 - 3x^2)\,dx + \int_{y_0}^{y} 2x_0 y dy = 0$$

$$[y^2x - x^3 \Big|_{x_0}^{x} + [x_0 y^2 \Big|_{y_0}^{y} = 0$$

$$y^2x - x^3 - y^2x_0 + {x_0}^3 + x_0y^2 - x_0{y_0}^2 = 0$$

$$y^2x - x^3 = C$$

$$(x+y)dx-(x-y)dy=0$$

$$\begin{cases} P_{(x,y)} = x+y & \rightarrow \quad \dfrac{\partial P}{\partial y} = 1 \\ Q_{(x,y)} = y-x & \rightarrow \quad \dfrac{\partial Q}{\partial x} = -1 \end{cases} \quad => \quad \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x} \quad => \quad \textit{not exact}$$

$$(x+y)dx-(x-y)dy=0 \quad => \quad \begin{cases} P_{(x,y)} = x+y \ \rightarrow \ P_{(tx,ty)} = tx+ty = t(x+y) = tP_{(x,y)} \\ Q_{(x,y)} = x-y \ \rightarrow \ O_{(tx,ty)} = tx-ty = t(x-y) = tQ_{(x,y)} \end{cases}$$

$$\begin{cases} P_{(1,z)} = 1+z \\ Q_{(1,z)} = 1-z \end{cases} \quad => \quad F_{(z)} = \frac{P_{(1,z)}}{Q_{(1,z)}} = -\frac{z+1}{z-1} \quad => \quad F_{(z)} - z = -\left(\frac{z+1}{z-1} + z\right) = -\frac{z^2+1}{z-1}$$

$$\frac{1}{F_{(z)} - z} = -\frac{z-1}{z^2+1} = -\left(\frac{z}{z^2+1} - \frac{1}{z^2+1}\right) = \frac{1}{z^2+1} - \frac{z}{z^2+1}$$

$$P(x,y)\,dx - Q(x,y)\,dy = 0$$

מש' דיפ' הומוגנית:

נקראת הומוגנית אם P ו Q הומוגניות מאותה מעלה.

$$P(tx,ty) = t^{\overset{\text{מעלה}}{n}} P(x,y)$$

$$F(z) = \frac{P(1,z)}{Q(1,z)}, \quad z = y/x$$

הפתרון:

$$\int \frac{dz}{F(z)-z} = \ln xc$$

$$\int \left(\frac{1}{z^2+1} - \frac{1}{2}\cdot\frac{2z}{z^2+1}\right)dz = \ln xc \quad => \quad \arctan(z) - \frac{1}{2}\ln(z^2+1) = \ln xc \quad =>$$

$$\arctan\left(\frac{y}{x}\right) - \frac{1}{2}\ln\left(\frac{y^2}{x^2}+1\right) = \ln x + \ln C \quad => \quad \arctan\left(\frac{y}{x}\right) - \left[\frac{1}{2}\ln\frac{x^2+y^2}{x^2} + \ln x\right] = \ln C$$

$$\arctan\left(\frac{y}{x}\right) - \left[\ln\frac{\sqrt{x^2+y^2}}{x} + \ln x\right] = \ln C$$

$$\arctan\left(\frac{y}{x}\right) - \ln\left(\frac{\sqrt{x^2+y^2}}{x}\cdot x\right) = \ln C$$

$$\arctan\left(\frac{y}{x}\right) - \ln\sqrt{x^2+y^2} = \ln C$$