$$\int_0^{\pi} \sin^2\frac{x}{4}\cos\frac{x}{4}\,dx \quad \rightarrow \quad \begin{matrix} u = \sin\left(\frac{x}{4}\right) \\ du = \frac{1}{4}\cos\left(\frac{x}{4}\right)dx \end{matrix}$$

$$\int_0^{\pi} sin^2\left(\frac{x}{4}\right)\cos\left(\frac{x}{4}\right)dx \;=\; 4\int_0^{\pi} sin^2\left(\frac{x}{4}\right)\cdot\frac{1}{4}\cos\left(\frac{x}{4}\right)dx \;=\; 4\int_0^{\sqrt{2}/2} u^2du \;=$$

$$= \frac{4}{3}\left[u^3\right|_0^{\sqrt{2}/2} \;=\; \frac{4}{3}\left(\frac{\sqrt{2}}{4}-0\right) \;=\; \frac{\sqrt{2}}{3}$$

$$\int_{\pi}^{\frac{3\pi}{2}}\left(ctg^5\left(\frac{\theta}{6}\right)\left(\frac{1}{\cos^2\left(\frac{\theta}{6}\right)}\right)\right)d\theta \;=\; \int_{\pi}^{\frac{3\pi}{2}}\frac{1}{tg^5\left(\frac{\theta}{6}\right)}\cdot\frac{1}{cos^2\left(\frac{\theta}{6}\right)}d\theta \quad \rightarrow \quad \begin{matrix} u = \tan\left(\frac{\theta}{6}\right) \\ du = \frac{1}{6}\frac{1}{cos^2\left(\frac{\theta}{6}\right)}d\theta \end{matrix}$$

$$\int_{\pi}^{\frac{3\pi}{2}}\frac{1}{tg^5\left(\frac{\theta}{6}\right)}\cdot\frac{1}{cos^2\left(\frac{\theta}{6}\right)}d\theta \;=\; 6\int_{\pi}^{\frac{3\pi}{2}}\frac{1}{tg^5\left(\frac{\theta}{6}\right)}\cdot\frac{1}{6}\frac{1}{cos^2\left(\frac{\theta}{6}\right)}d\theta \;=\; 6\int_{\frac{\sqrt{3}}{3}}^{1}\frac{1}{u^5}du \;=\; 6\int_{\frac{\sqrt{3}}{3}}^{1}u^{-5}du \;=$$

$$= \frac{6}{-4}\left[u^{-4}\right|_{\sqrt{3}/3}^{1} \;=\; -\frac{3}{2}\left[\frac{1}{u^4}\right|_{\sqrt{3}/3}^{1} \;=\; -\frac{3}{2}(1-9) \;=\; 12$$

$$\int\frac{1+x^2}{1+x^4}dx$$

$$\frac{1+x^2}{1+x^4} = \frac{1+x^2}{1+x^4}\cdot\frac{x^{-2}}{x^{-2}} = \frac{x^{-2}+1}{x^{-2}+x^2} = \frac{1+x^{-2}}{x^2+x^{-2}} = \frac{1+x^{-2}}{(x-x^{-1})^2+2}$$

$$(x-x^{-1})^2 = x^2-2+x^{-2} \quad => \quad (x-x^{-1})^2+2 = x^2+x^{-2}$$

$$\frac{1+x^2}{1+x^4}dx \;=\; \frac{1+x^{-2}}{(x-x^{-1})^2+2}dx \quad\rightarrow\quad \begin{matrix} u = x-x^{-1} \\ du = (1+x^{-2})dx \end{matrix} \;=\; \frac{du}{u^2+2}$$

$$\int\frac{1+x^2}{1+x^4}dx \;=\; \int\frac{du}{u^2+2} \;=\; \frac{1}{2}\int\frac{du}{\left(\frac{u}{\sqrt{2}}\right)^2+1} \quad\rightarrow\quad \begin{matrix} t = \frac{u}{\sqrt{2}} \\ dt = \frac{1}{\sqrt{2}}du \end{matrix} \;=\; \frac{\sqrt{2}}{2}\int\frac{\frac{1}{\sqrt{2}}du}{\left(\frac{u}{\sqrt{2}}\right)^2+1} \;=$$

$$= \frac{\sqrt{2}}{2}\int\frac{dt}{t^2+1} \;=\; \frac{\sqrt{2}}{2}arctg(t)+C \;=\; \frac{\sqrt{2}}{2}arctg\left(\frac{u}{\sqrt{2}}\right)+C \;=\; \frac{\sqrt{2}}{2}arctg\left(\frac{x-x^{-1}}{\sqrt{2}}\right)+C$$