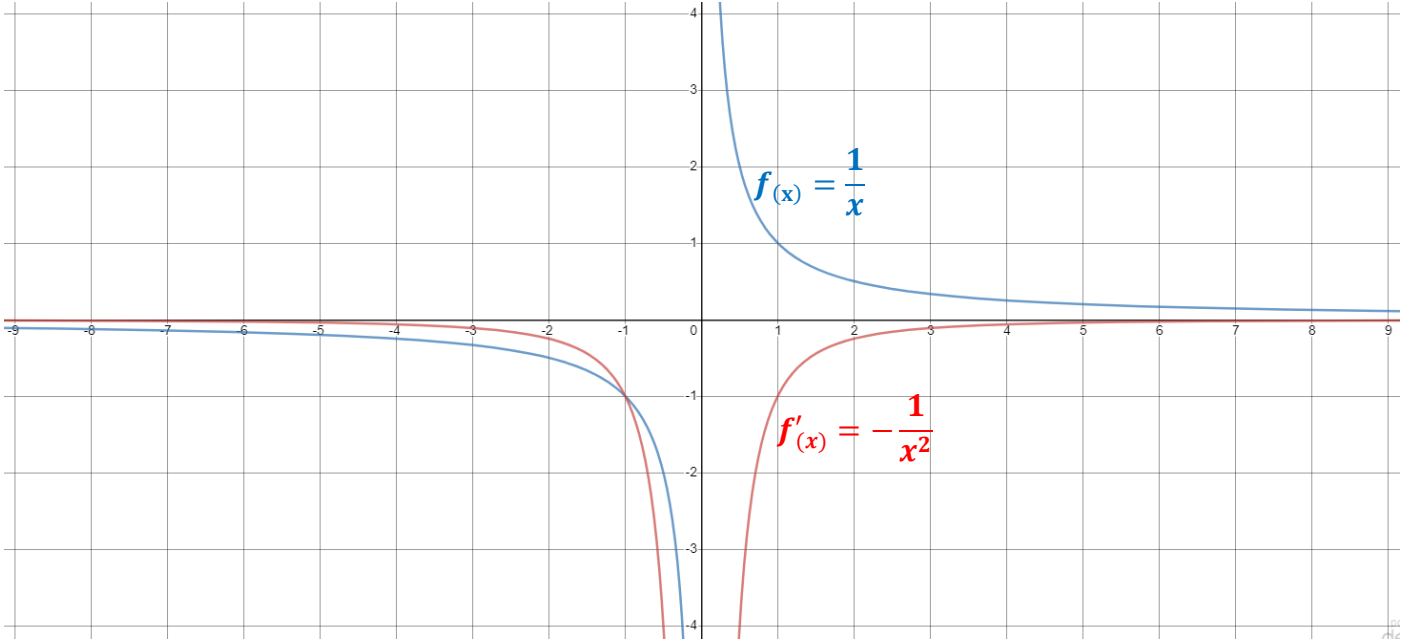


חשב את הנגזרת לפונקציה $f(x) = \frac{1}{x}$ על פי הגדרת הנגזרת.

פיתרון:

$$\begin{aligned} f'(x) &= \frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{1}{\Delta x} \cdot \frac{x - (x + \Delta x)}{x(x + \Delta x)} \right] = \\ &= \lim_{\Delta x \rightarrow 0} \left[\frac{1}{\Delta x} \cdot \frac{-\Delta x}{x(x + \Delta x)} \right] = -\frac{1}{x} \lim_{\Delta x \rightarrow 0} \left[\frac{1}{\Delta x} \cdot \frac{\Delta x}{(x + \Delta x)} \right] = -\frac{1}{x} \lim_{\Delta x \rightarrow 0} \left[\frac{1}{(x + \Delta x)} \right] = -\frac{1}{x} \left(\frac{1}{x} \right) = -\frac{1}{x^2} \end{aligned}$$



חשב את הנגזרת לפונקציה $f(x) = \sin x$ על פי הגדרת הנגזרת.

פיתרון:

$$\begin{aligned} f'(x) &= \lim_{\Delta x \rightarrow 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\sin(x + \Delta x) - \sin x}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x} \right] = \\ &= \lim_{\Delta x \rightarrow 0} \left[\frac{\sin x (\cos \Delta x - 1) + \cos x \sin \Delta x}{\Delta x} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\cos \Delta x - 1}{\Delta x} \sin x + \frac{\sin \Delta x}{\Delta x} \cos x \right] = \\ &= \lim_{\Delta x \rightarrow 0} \left[\frac{\cos \Delta x - 1}{\Delta x} \sin x \right] + \lim_{\Delta x \rightarrow 0} \left[\frac{\sin \Delta x}{\Delta x} \cos x \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{\cos \Delta x - 1}{\Delta x} \right] \sin x + \lim_{\Delta x \rightarrow 0} \left[\frac{\sin \Delta x}{\Delta x} \right] \cdot \cos x = \\ &0 \cdot \sin x + 1 \cdot \cos x = \cos x \end{aligned}$$

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \left[\frac{\cos \Delta x - 1}{\Delta x} \right] &= \lim_{\Delta x \rightarrow 0} \left[\frac{\cos^2 \Delta x - 1}{\Delta x (\cos \Delta x + 1)} \right] = \lim_{\Delta x \rightarrow 0} \left[\frac{-\sin^2 \Delta x}{\Delta x (\cos \Delta x + 1)} \right] = -\lim_{\Delta x \rightarrow 0} \left[\frac{\sin \Delta x}{\Delta x} \cdot \frac{\sin \Delta x}{\cos \Delta x + 1} \right] = \\ &= -\lim_{\Delta x \rightarrow 0} \left[\frac{\sin \Delta x}{\Delta x} \right] \cdot \lim_{\Delta x \rightarrow 0} \left[\frac{\sin \Delta x}{\cos \Delta x + 1} \right] = -1 \cdot \frac{0}{2} = 0 \end{aligned}$$