$$\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{1}{lnx}} = [0^0] = ?$$

$$y = \left(\frac{1}{x}\right)^{\frac{1}{lnx}} = (x^{-1})^{\frac{1}{lnx}} = x^{-\frac{1}{lnx}} \quad => \quad \ln y = \ln x^{-\frac{1}{lnx}} = -\frac{1}{lnx}\ln x = -1$$

$$y = e^{\ln y} = e^{-1} = \frac{1}{e}$$

$$\lim_{x\to\infty}\left(\frac{1}{x}\right)^{\frac{1}{lnx}} = \lim_{x\to\infty} y = \lim_{x\to\infty}\frac{1}{e} = \frac{1}{e}$$

$$\lim_{x\to 0}\frac{\tan(\sin x)}{\sin(\tan x)} = \lim_{x\to 0}\frac{\tan(x)}{\sin(x)} = \lim_{x\to 0}\frac{x}{x} = \lim_{x\to 0} 1 = 1$$

$When\ x \to 0 \quad , \quad \sin x \to x \quad and\ also \quad \tan x \to x$

$$\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{t\to 0}\frac{\sin(t)}{t} = 1$$

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right) = 0\cdot bounded = 0$$

$$\lim_{x\to\infty}\left(\frac{1-x}{2-x}\right)^{\frac{3}{x}} = [1^0] => Euler$$

$$\left(\frac{1-x}{2-x}\right)^{\frac{3}{x}} = \left(\frac{2-x-1}{2-x}\right)^{\frac{3}{x}} = \left(1-\frac{1}{2-x}\right)^{\frac{3}{x}} = \left(1+\frac{1}{x-2}\right)^{\frac{3}{x}} = \left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\left(\frac{1}{x-2}\right)\frac{3}{x}} = \left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\frac{3}{x^2-2x}}$$

$$\lim_{x\to\infty}\left(\frac{1-x}{2-x}\right)^{\frac{3}{x}} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\frac{3}{x^2-2x}} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\lim_{x\to\infty}\frac{3}{x^2-2x}} = e^0 = 1$$

$$\lim_{x\to\infty}\left(\frac{1-x}{2-x}\right)^{\frac{x}{3}} = [1^\infty] => Euler$$

$$\left(\frac{1-x}{2-x}\right)^{\frac{x}{3}} = \left(\frac{2-x-1}{2-x}\right)^{\frac{x}{3}} = \left(1-\frac{1}{2-x}\right)^{\frac{x}{3}} = \left(1+\frac{1}{x-2}\right)^{\frac{x}{3}} = \left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\left(\frac{1}{x-2}\right)\frac{x}{3}} = \left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\frac{x}{3x-6}}$$

$$\lim_{x\to\infty}\left(\frac{1-x}{2-x}\right)^{\frac{3}{x}} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\frac{x}{3x-6}} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x-2}\right)^{x-2}\right]^{\lim_{x\to\infty}\frac{x}{3x-6}} = e^{1/3}$$

$$\lim_{x\to\infty} \sqrt[x]{2^x+3^x} = \lim_{x\to\infty} (2^x+3^x)^{1/x} = [\infty^0] \ => \ Euler$$

$$\sqrt[x]{2^x+3^x} \ = \ (2^x+3^x)^{1/x} \ = \ \left\{3^x\left[\left(\frac{2}{3}\right)^x+1\right]\right\}^{1/x} = \ 3\left[1+\left(\frac{2}{3}\right)^x\right]^{1/x} = \ 3\left\{\left[1+\left(\frac{2}{3}\right)^x\right]^{\left(\frac{3}{2}\right)^x}\right\}^{\left(\frac{2}{3}\right)^x\cdot\frac{1}{x}}$$

$$\lim_{x\to\infty} \sqrt[x]{2^x+3^x} = \lim_{x\to\infty} 3\left\{\left[1+\left(\frac{2}{3}\right)^x\right]^{\left(\frac{3}{2}\right)^x}\right\}^{\left(\frac{2}{3}\right)^x\cdot\frac{1}{x}} = \ 3\lim_{x\to\infty}\left\{\left[1+\left(\frac{2}{3}\right)^x\right]^{\left(\frac{3}{2}\right)^x}\right\}^{\lim_{x\to\infty}\left(\frac{2}{3}\right)^x\cdot\frac{1}{x}} =$$

$$= \ 3\lim_{x\to\infty}\left\{\left[1+\left(\frac{2}{3}\right)^x\right]^{\left(\frac{3}{2}\right)^x}\right\}^{\lim_{x\to\infty}\left(\frac{2}{3}\right)^x\cdot\lim_{x\to\infty}\cdot\frac{1}{x}\cdot} = \ 3e^0 \ = \ 3$$

$$\lim_{x\to\infty}\left(\frac{2x+3}{5x+4}\right)^{2x} = \lim_{x\to\infty}\left(\frac{2x+3}{5x+4}\right)^{\lim_{x\to\infty}\cdot 2x\cdot} = \left(\frac{2}{5}\right)^{\infty} = \ 0$$