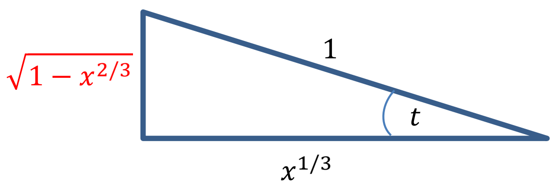
$$\int_{1/8}^{1}\left[x-\frac{1}{3}\left(1-x^{2/3}\right)^{3/2}\right]dx \;=\; \int_{1/8}^{1}\left[x-\frac{1}{3}\sqrt{1-x^{2/3}}^{\,3}\right]dx \;=\; \int_{1/8}^{1} x\,dx-\frac{1}{3}\int_{\frac{1}{8}}^{1}\sqrt{1-x^{\frac{2}{3}}}^{\,3}\,dx$$

$$-\frac{1}{3}\int\sqrt{1-x^{\frac{2}{3}}}^{\,3}\,dx =$$



$$\sin t=\sqrt{1-x^{2/3}} \quad => \quad sin^3 t=\sqrt{1-x^{2/3}}^{\,3}$$

$$x^{1/3}=\cos t \quad => \quad x=cos^3 t \quad => \quad dx=-3cos^2 t\cdot\sin t\,dt$$

$$= -\frac{1}{3}\int sin^3 t\,(-3)cos^2 t\cdot\sin t\cdot dt \;=\; \int sin^4 t\,cos^2 t\cdot dt=\int sin^4 t\,(1-sin^2 t)\cdot dt=\int (sin^4 t-sin^6 t)\cdot dt \;=\; ....$$

$$\cos 2\alpha=1-2sin^2\alpha \quad => \quad sin^2\alpha=\frac{1}{2}(1-\cos 2\alpha)$$

$$\cos 2\alpha=2cos^2\alpha-1 \quad => \quad cos^2\alpha=\frac{1}{2}(1+\cos 2\alpha)$$

$$*\int sin^4 t\cdot dt \;=\; \int (sin^2 t)^2\cdot dt \;=\; \frac{1}{4}\int[1-\cos(2t)]^2\cdot dt \;=\; \frac{1}{4}\int[1-2\cos(2t)+cos^2(2t)]\cdot dt \;=$$

$$=\frac{1}{4}\int\left[1-2\cos(2t)+\frac{1}{2}(1+\cos(4t))\right]\cdot dt \;=\; \frac{1}{4}\int\left[\frac{3}{2}-2\cos(2t)+\frac{1}{2}\cos(4t)\right]\cdot dt \;=$$

$$\frac{1}{8}\int[3-4\cos(2t)+\cos(4t)]\cdot dt \;=\; \frac{1}{8}\left[3t-2\sin(2t)+\frac{1}{4}\sin(4t)\right]$$

$$*\int sin^6 t\cdot dt=\int (sin^2 t)^3\cdot dt \;=\; \frac{1}{8}\int[1-\cos(2t)]^3\cdot dt \;=\; \frac{1}{8}\int[1-3\cos(2t)+3cos^2(2t)-cos^3(2t)]\cdot dt \;=$$

$$=\frac{1}{8}\int\left[1-3\cos(2t)+\frac{3}{2}(1+\cos(4t))-\cos(2t)\left(1-sin^2(2t)\right)\right]\cdot dt \;=$$

$$=\frac{1}{8}\int\left[1-3\cos(2t)+\frac{3}{2}+\frac{3}{2}\cos(4t)-\cos(2t)+sin^2(2t)\cdot\cos(2t)\right]\cdot dt \;=$$

$$=\frac{1}{8}\int\left[\frac{5}{2}-4\cos(2t)+\frac{3}{2}\cos(4t)+\frac{1}{2}sin^2(2t)\cdot 2\cos(2t)\right]\cdot dt \;=$$

$$=\frac{1}{8}\left[\frac{5}{2}t-2\sin(2t)+\frac{3}{8}\sin(4t)+\frac{1}{6}sin^3(2t)\right]$$

$$\dots = \int (sin^4 t - sin^6 t)\cdot dt = \frac{1}{8}\left[3t - 2\sin(2t) + \frac{1}{4}\sin(4t) - \frac{5}{2}t + 2\sin(2t) - \frac{3}{8}\sin(4t) - \frac{1}{6}sin^3(2t)\right] =$$

$$= \frac{1}{8}\left[\frac{1}{2}t - \frac{1}{8}\sin(4t) - \frac{1}{6}sin^3(2t)\right] \quad => \quad \int (sin^6 t - sin^4 t)\cdot dt = \frac{1}{8}\left[\frac{1}{8}\sin(4t) - \frac{1}{2}t + \frac{1}{6}sin^3(2t)\right]$$

$$\int_{1/8}^{1}\left[x - \frac{1}{3}\left(1 - x^{2/3}\right)^{3/2}\right]dx = \int_{1/8}^{1}\left[x - \frac{1}{3}\sqrt{1 - x^{2/3}}^{\,3}\right]dx = \int_{1/8}^{1} x\,dx - \frac{1}{3}\int_{\frac{1}{8}}^{1}\sqrt{1 - x^{\frac{2}{3}}}^{\,3}\,dx =$$

$$x = cos^3 t \quad => \quad \begin{cases} x = \frac{1}{8} \rightarrow t = \pi/3 \\ x = 1 \rightarrow t = 0 \end{cases}$$

$$= \int_{\frac{1}{8}}^{1} x\,dx + \int_{\pi/3}^{0} (sin^4 t - sin^6 t)\,dt = \int_{\frac{1}{8}}^{1} x\,dx + \int_{0}^{\pi/3} (sin^6 t - sin^4 t)\,dt =$$

$$= \frac{1}{2}x^2\,\Big|_{1/8}^{1} + \frac{1}{8}\left[\frac{1}{8}\sin(4t) - \frac{1}{2}t + \frac{1}{6}sin^3(2t)\right]_0^{\frac{\pi}{3}} =$$

$$= \frac{1}{2}\left(1 - \frac{1}{64}\right) + \frac{1}{8}\left[\frac{1}{8}\sin\left(\frac{4\pi}{3}\right) - \frac{\pi}{6} + \frac{1}{6}sin^3\left(\frac{2\pi}{3}\right) - (0 - 0 - 0 + 0)\right] =$$

$$= \frac{63}{128} + \frac{1}{8}\left[-\frac{\sqrt{3}}{16} - \frac{\pi}{6} + \frac{\sqrt{3}}{16}\right] = \frac{63}{128} - \frac{\pi}{48} = 0.426738$$