$$\int_1^{\infty}\left(\frac{ax}{1+x^2}-\frac{1}{2x}\right)dx \quad for\ what\ values\ of\ a\ the\ integral\ Converges?$$

$$\frac{ax}{1+x^2}-\frac{1}{2x} = \frac{2ax^2-1-x^2}{2x(1+x^2)} = \frac{(2a-1)x^2-1}{2x(1+x^2)}$$

$$for\ x\to\infty\ \ \frac{(2a-1)x^2-1}{2x(1+x^2)}\ can\ be\ replaced\ by\ the\ roof: \frac{(2a-1)x^2}{2x(x^2)} = \frac{2a-1}{2x}$$

$$\frac{2a-1}{2}\int_1^{\infty}\frac{1}{x}dx = \frac{2a-1}{2}\lim_{b\to\infty}\int_1^{b}\frac{1}{x}dx = \frac{2a-1}{2}\lim_{b\to\infty}[lnx\Big|_1^b =$$

$$= \frac{2a-1}{2}\lim_{b\to\infty}[\ln(b)-ln1] = \frac{2a-1}{2}\lim_{b\to\infty}\ln(b)$$

כידוע, $\lim_{b\to\infty}\ln(b)=\infty$ ולכן תוצאה סופית (אפס בהכרח) תתקבל רק אם נכפול את הגבול הזה באפס:

$$\frac{2a-1}{2}\cdot\lim_{b\to\infty}\ln(b) = 0\ (finite) \quad => \quad \frac{2a-1}{2}=0 \quad => \quad a=\frac{1}{2}$$

כעת לחישוב האינטגרל כאשר $a=\frac{1}{2}$:

$$\int_1^{\infty}\left(\frac{ax}{1+x^2}-\frac{1}{2x}\right)dx \quad , \quad a=\frac{1}{2} \quad => \quad \int_1^{\infty}\left(\frac{1}{2}\cdot\frac{x}{1+x^2}-\frac{1}{2x}\right)dx$$

$$\frac{1}{2}\int\frac{x}{1+x^2}dx = \frac{1}{4}\int\frac{2x}{1+x^2}dx = \frac{1}{4}\int\frac{du}{u} = \frac{1}{4}\ln(u) = \frac{1}{4}\ln(1+x^2)$$

$$\frac{1}{2}\int\frac{1}{x}dx = \frac{1}{2}\ln(x)$$

$$\int\left(\frac{1}{2}\cdot\frac{x}{1+x^2}-\frac{1}{2x}\right)dx = \frac{1}{4}ln(1+x^2)-\frac{1}{2}\ln(x) = ln(1+x^2)^{\frac{1}{4}}-ln(x)^{\frac{1}{2}} = ln\frac{(1+x^2)^{\frac{1}{4}}}{x^{\frac{1}{2}}}$$

$$\int_1^{\infty}\left(\frac{1}{2}\cdot\frac{x}{1+x^2}-\frac{1}{2x}\right)dx = \lim_{b\to\infty}\int_1^{b}\left(\frac{1}{2}\cdot\frac{x}{1+x^2}-\frac{1}{2x}\right)dx = \lim_{b\to\infty}\left[ln\frac{(1+x^2)^{\frac{1}{4}}}{x^{\frac{1}{2}}}\right|_1^b =$$

$$= \lim_{b\to\infty}\left[ln\frac{(1+b^2)^{\frac{1}{4}}}{b^{\frac{1}{2}}}-ln\frac{2^{\frac{1}{4}}}{1}\right] = \lim_{b\to\infty}\left[ln\frac{(1+b^2)^{\frac{1}{4}}}{2^{\frac{1}{4}}b^{\frac{1}{2}}}\right] = \lim_{b\to\infty}\left[ln\frac{(b^2)^{\frac{1}{4}}}{2^{\frac{1}{4}}b^{\frac{1}{2}}}\right] =$$

$$= \lim_{b\to\infty}\left[ln\frac{b^{\frac{1}{2}}}{2^{\frac{1}{4}}b^{\frac{1}{2}}}\right] = ln\frac{1}{2^{\frac{1}{4}}} = -\ln 2^{\frac{1}{4}} = -\frac{ln2}{4}$$