

חשב את האינטגרל הבא:

$$\int_{-1}^4 \frac{dx}{\sqrt{|x|}} = \lim_{b \rightarrow 0^-} \int_{-1}^b \frac{dx}{\sqrt{-x}} + \lim_{a \rightarrow 0^+} \int_a^4 \frac{dx}{\sqrt{x}}$$

$$\int_{-1}^b \frac{dx}{\sqrt{-x}} \rightarrow \begin{matrix} u = -x \\ du = -dx \end{matrix} = - \int_{-1}^b \frac{-dx}{\sqrt{-x}} = - \int_1^{-b} \frac{du}{\sqrt{u}} = \int_{-b}^1 \frac{du}{\sqrt{u}} = 2 \int_{-b}^1 \frac{du}{2\sqrt{u}} = 2[\sqrt{u}]_{-b}^1 = 2(1 - \sqrt{-b})$$

$$\lim_{b \rightarrow 0^-} \int_{-1}^b \frac{dx}{\sqrt{-x}} = 2 \lim_{b \rightarrow 0^-} (1 - \sqrt{-b}) = 2(1 - \sqrt{0^+}) = 2$$

$$\int_a^4 \frac{dx}{\sqrt{x}} = 2 \int_a^4 \frac{dx}{2\sqrt{x}} = 2[\sqrt{x}]_a^4 = 2(2 - \sqrt{a})$$

$$\lim_{a \rightarrow 0^+} \int_a^4 \frac{dx}{\sqrt{x}} = 2 \lim_{a \rightarrow 0^+} (2 - \sqrt{a}) = 2(2 - \sqrt{0^+}) = 4$$

לסיכום

$$\int_{-1}^4 \frac{dx}{\sqrt{|x|}} = \lim_{b \rightarrow 0^-} \int_{-1}^b \frac{dx}{\sqrt{-x}} + \lim_{a \rightarrow 0^+} \int_a^4 \frac{dx}{\sqrt{x}} = 2 + 4 = 6$$

חשב את האינטגרל הבא:

$$\int_{-\infty}^0 \theta e^\theta d\theta = \lim_{a \rightarrow -\infty} \int_a^0 \theta e^\theta d\theta$$

$$\int \theta e^\theta d\theta \rightarrow \begin{matrix} u = \theta \\ dv = e^\theta d\theta \end{matrix} \Rightarrow \begin{matrix} du = d\theta \\ v = e^\theta \end{matrix} = \theta e^\theta - \int e^\theta d\theta = \theta e^\theta - e^\theta$$

$$\int_a^0 \theta e^\theta d\theta = [\theta e^\theta - e^\theta]_a^0 = 0 - 1 - (ae^a - e^a) = e^a - ae^a - 1$$

$$\lim_{a \rightarrow -\infty} \int_a^0 \theta e^\theta d\theta = \lim_{a \rightarrow -\infty} (e^a - ae^a - 1) = \lim_{a \rightarrow -\infty} (e^a) - \lim_{a \rightarrow -\infty} (ae^a) - \lim_{a \rightarrow -\infty} (1) = 0 - ? - 1$$

$$\lim_{a \rightarrow -\infty} (ae^a) = [-\infty \cdot 0] = \lim_{a \rightarrow -\infty} \frac{a}{1/e^a} = \left[\frac{-\infty}{\infty} \right] = \lim_{a \rightarrow -\infty} \frac{1}{-1/e^{2a}} e^a = - \lim_{a \rightarrow -\infty} e^{3a} = 0$$

לסיכום

$$\int_{-\infty}^0 \theta e^\theta d\theta = \lim_{a \rightarrow -\infty} \int_a^0 \theta e^\theta d\theta = \lim_{a \rightarrow -\infty} (e^a - ae^a - 1) = 0 - 0 - 1 = -1$$