$$\int_{1}^{4}\left(\frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}\right)$$

$$\int_{1}^{4}\frac{1}{(1+\sqrt{y})^2}\cdot\frac{dy}{2\sqrt{y}} \quad\rightarrow\quad \begin{array}{l} u = 1+\sqrt{y} \\ du = \dfrac{dy}{2\sqrt{y}} \end{array} \quad=\quad \int_{2}^{3}\frac{1}{u^2}\cdot du \quad=\quad -\frac{1}{u}\Big|_{2}^{3} \quad=\quad -\left(\frac{1}{3}-\frac{1}{2}\right) \quad=\quad \frac{1}{6}$$

$$\int\sqrt{\frac{x-1}{x^5}}\,dx$$
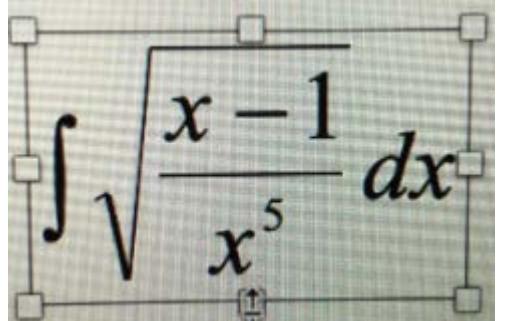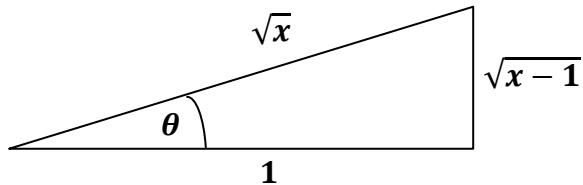
$$\sin\theta = \sqrt{\frac{x-1}{x}} \quad,\quad \cos\theta = \frac{1}{\sqrt{x}} \quad,\quad \tan\theta = \sqrt{x-1}$$

$$\frac{1}{\sqrt{x}} = \cos\theta \;=> \; \frac{1}{x} = cos^2\theta \;=> \; x = \frac{1}{cos^2\theta} \;=> \; \frac{dx}{d\theta} = \frac{2\cos\theta\sin\theta}{cos^4\theta} \;=> \; dx = \frac{2\sin\theta}{cos^3\theta}\,d\theta$$

$$\sqrt{\frac{x-1}{x^5}}\,dx \;=\; \sqrt{x-1}\cdot\frac{1}{\sqrt{x^5}}\,dx \;=\; \sqrt{x-1}\cdot\left(\frac{1}{\sqrt{x}}\right)^5 dx \;=\; \tan\theta\cdot cos^5\theta\cdot\frac{2\sin\theta}{cos^3\theta}\,d\theta \;=\; 2sin^2\theta\cos\theta\,d\theta$$

$$\sqrt{\frac{x-1}{x^5}}\,dx \;=\; 2sin^2\theta\cos\theta\,d\theta \quad\rightarrow\quad \begin{array}{l} u = \sin\theta \\ du = \cos\theta\,d\theta \end{array}$$

$$\int\sqrt{\frac{x-1}{x^5}}\,dx \;=\; 2\int sin^2\theta\cos\theta\,d\theta \;=\; 2\int u^2 du \;=\; \frac{2}{3}u^3 + C \;=\; \frac{2}{3}sin^3\theta + C \;=\; \frac{2}{3}\sqrt{\frac{x-1}{x}}^{\,3} + C$$

בדיקה:

$$f_{(x)} = \frac{2}{3}\left(1-\frac{1}{x}\right)^{3/2} + C \quad=> \quad f'_{(x)} = \left(1-\frac{1}{x}\right)^{1/2}\frac{1}{x^2} \;=\; \sqrt{\frac{x-1}{x}}\sqrt{\frac{1}{x^4}} \;=\; \sqrt{\frac{x-1}{x^5}}$$