

$$\int \frac{dx}{\sqrt{5x+8}}$$

$$\begin{aligned} \int \frac{dx}{\sqrt{5x+8}} &\Rightarrow \begin{cases} u = 5x+8 \\ du = 5dx \end{cases} = \frac{1}{5} \int \frac{5dx}{\sqrt{5x+8}} = \\ &= \frac{1}{5} \int \frac{du}{\sqrt{u}} = \frac{2}{5} \int \frac{du}{2\sqrt{u}} = \frac{2}{5} \sqrt{u} + C = \frac{2}{5} \sqrt{5x+8} + C \end{aligned}$$

$$\int \frac{dx}{\cos x}$$

$$\begin{aligned} \int \frac{1}{\cos(x)} dx &= \int \frac{\cos(x)}{\cos^2(x)} dx = \int \frac{\cos(x)}{1-\sin^2(x)} dx \rightarrow \begin{cases} u = \sin(x) \\ du = \cos(x) dx \end{cases} = \\ &= \int \frac{du}{1-u^2} = \int \frac{1}{(1-u)(1+u)} du = \dots \\ \frac{1}{(1-u)(1+u)} &= \frac{A}{1-u} + \frac{B}{1+u} = \frac{A(1+u) + B(1-u)}{(1-u)(1+u)} = \frac{(A-B)u + A+B}{(1-u)(1+u)} \Rightarrow \\ \Rightarrow \begin{cases} A-B=0 \\ A+B=1 \end{cases} &\Rightarrow A=B=\frac{1}{2} \Rightarrow \frac{1}{(1-u)(1+u)} = \frac{1/2}{1-u} + \frac{1/2}{1+u} = \frac{1}{2} \left( \frac{1}{1-u} + \frac{1}{1+u} \right) \\ \dots &= \frac{1}{2} \int \left( \frac{1}{1-u} + \frac{1}{1+u} \right) du = \frac{1}{2} (-\ln|1-u| + \ln|1+u|) + C = \\ &= \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C = \ln \sqrt{\frac{1+u}{1-u}} + C = \ln \sqrt{\frac{1+\sin(x)}{1-\sin(x)}} + C \end{aligned}$$