

6. מצאו את הפונקציות המקיימות :

$$\frac{dr}{d\theta} = 3\cos^2\left(\frac{\pi}{4} - \theta\right); \quad r(0) = \frac{\pi}{8}$$

$$\cos(2\alpha) = 2\cos^2(\alpha) - 1 \quad \Rightarrow \quad \cos^2(\alpha) = \frac{1}{2}[\cos(2\alpha) + 1]$$

$$\int \cos^2(x) dx = \frac{1}{2} \int [\cos(2x) + 1] dx$$

$$dr = 3\cos^2\left(\theta - \frac{\pi}{4}\right) d\theta \quad \Rightarrow \quad \int dr = \int 3\cos^2\left(\theta - \frac{\pi}{4}\right) d\theta$$

$$r(\theta) = 3 \int \cos^2\left(\theta - \frac{\pi}{4}\right) d\theta = \frac{3}{2} \int \left[\cos\left(2\theta - \frac{\pi}{2}\right) + 1\right] d\theta = \frac{3}{2} \left[\frac{\sin\left(2\theta - \frac{\pi}{2}\right)}{2} + \theta \right] + C$$

$$r(0) = \frac{\pi}{8} \quad \Rightarrow \quad \frac{\pi}{8} = \frac{3}{2} \left[\frac{\sin\left(2 \cdot 0 - \frac{\pi}{2}\right)}{2} + 0 \right] + C \quad \Rightarrow \quad \frac{\pi}{8} = \frac{3}{2} \cdot \frac{\sin\left(-\frac{\pi}{2}\right)}{2} + C \quad \Rightarrow$$

$$\Rightarrow \quad \frac{\pi}{8} = \frac{3}{2} \cdot \frac{-1}{2} + C \quad \Rightarrow \quad C = \frac{\pi}{8} + \frac{3}{4} = \frac{\pi + 6}{8}$$

$$r(\theta) = \frac{3}{2} \left[\frac{\sin\left(2\theta - \frac{\pi}{2}\right)}{2} + \theta \right] + \frac{\pi + 6}{8}$$