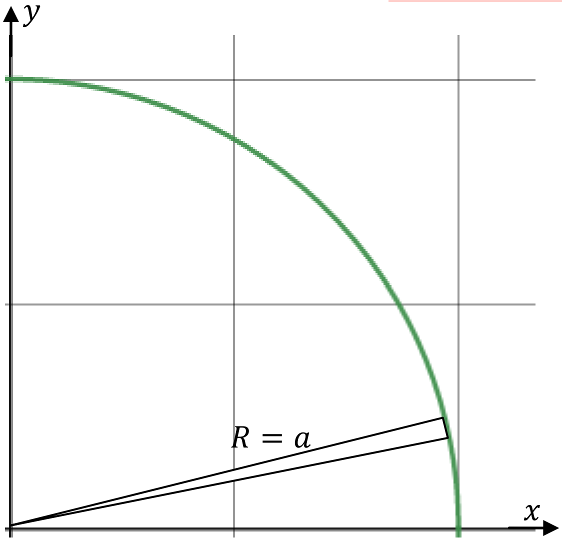
חשב את $\displaystyle\iint_Q e^{(x^2+y^2)}\,dA$ כאשר Q הוא רבע הדיסקה:

$a>0\,,\ a^2\ge x^2+y^2\,,\ y\ge 0,\ x\ge 0$



$$\int_0^{\pi/2}\int_0^{a} e^{r^2} r\,dr\,d\theta \quad\rightarrow\quad \begin{matrix} u = r^2 & 0<r<a \\ du = 2r\,dr & 0<u<a^2 \end{matrix}$$

$$\frac{1}{2}\int_0^{a} e^{r^2}\,2r\,dr = \frac{1}{2}\int_0^{a^2} e^{u}\,du = \frac{1}{2}e^{u}\Big|_0^{a^2} = \frac{1}{2}\left(e^{a^2}-1\right)$$

$$\frac{1}{2}\left(e^{a^2}-1\right)\int_0^{\pi/2} d\theta = \frac{1}{2}\left(e^{a^2}-1\right)\theta\Big|_0^{\pi/2} = \frac{\pi}{4}\left(e^{a^2}-1\right)$$

חשב את $\displaystyle\iint_D \sin\left(\frac{\pi x}{2y}\right)dx\,dy$ כאשר D הוא התחום ברביע הראשון המוגבל על ידי $y\ge x$,

$y\le\sqrt[3]{x}\,,\ y\ge\frac{1}{\sqrt{2}}$

$$\int_{\frac{1}{\sqrt{2}}}^{1}\int_{y^3}^{y}\sin\left(\frac{\pi}{2}\cdot\frac{x}{y}\right)dx\,dy$$

$$\int_{y^3}^{y}\sin\left(\frac{\pi}{2}\cdot\frac{x}{y}\right)dx = -\frac{2y}{\pi}\cos\left(\frac{\pi}{2}\cdot\frac{x}{y}\right)\Big|_{y^3}^{y} = \frac{2}{\pi}y\cos\left(\frac{\pi}{2}\cdot y^2\right)$$

$$\frac{2}{\pi}\int_{\frac{1}{\sqrt{2}}}^{1} y\cos\left(\frac{\pi}{2}\cdot y^2\right)dy \quad\rightarrow\quad \begin{matrix} u = \frac{\pi}{2}\cdot y^2 \\ du = \pi y\,dy \end{matrix} \quad = \frac{2}{\pi^2}\int_{\frac{1}{\sqrt{2}}}^{1}\cos\left(\frac{\pi}{2}\cdot y^2\right)\pi y\,dy$$

$$= \frac{2}{\pi^2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos(u)\,du = \frac{2}{\pi^2}\sin(u)\Big|_{\pi/4}^{\pi/2} = \frac{2}{\pi^2}\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{4}\right) =$$

$$= \frac{2}{\pi^2}\left(1-\frac{\sqrt{2}}{2}\right)$$