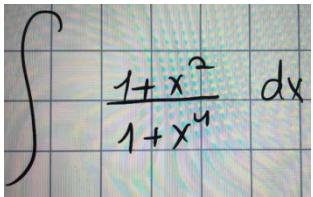


$$\int_0^\pi \sin^2 \frac{x}{4} \cos \frac{x}{4} dx \rightarrow u = \sin \left( \frac{x}{4} \right) \\ du = \frac{1}{4} \cos \left( \frac{x}{4} \right) dx$$

$$\int_0^\pi \sin^2 \left( \frac{x}{4} \right) \cos \left( \frac{x}{4} \right) dx = 4 \int_0^\pi \sin^2 \left( \frac{x}{4} \right) \cdot \frac{1}{4} \cos \left( \frac{x}{4} \right) dx = 4 \int_0^{\sqrt{2}/2} u^2 du = \\ = \frac{4}{3} \left[ u^3 \right]_{0}^{\sqrt{2}/2} = \frac{4}{3} \left( \frac{\sqrt{2}}{4} - 0 \right) = \frac{\sqrt{2}}{3}$$

$$\int_\pi^{\frac{3\pi}{2}} \left( \operatorname{ctg}^5 \left( \frac{\theta}{6} \right) \left( \frac{1}{\cos^2 \left( \frac{\theta}{6} \right)} \right) \right) d\theta = \int_\pi^{\frac{3\pi}{2}} \frac{1}{\operatorname{tg}^5 \left( \frac{\theta}{6} \right)} \cdot \frac{1}{\cos^2 \left( \frac{\theta}{6} \right)} d\theta \rightarrow u = \tan \left( \frac{\theta}{6} \right) \\ du = \frac{1}{6} \frac{1}{\cos^2 \left( \frac{\theta}{6} \right)} d\theta$$

$$\int_\pi^{\frac{3\pi}{2}} \frac{1}{\operatorname{tg}^5 \left( \frac{\theta}{6} \right)} \cdot \frac{1}{\cos^2 \left( \frac{\theta}{6} \right)} d\theta = 6 \int_\pi^{\frac{3\pi}{2}} \frac{1}{\operatorname{tg}^5 \left( \frac{\theta}{6} \right)} \cdot \frac{1}{6} \frac{1}{\cos^2 \left( \frac{\theta}{6} \right)} d\theta = 6 \int_{\frac{\sqrt{3}}{3}}^1 \frac{1}{u^5} du = 6 \int_{\frac{\sqrt{3}}{3}}^1 u^{-5} du = \\ = \frac{6}{-4} \left[ u^{-4} \right]_{\sqrt{3}/3}^1 = -\frac{3}{2} \left[ \frac{1}{u^4} \right]_{\sqrt{3}/3}^1 = -\frac{3}{2} (1 - 9) = 12$$



$$\frac{1+x^2}{1+x^4} = \frac{1+x^2}{1+x^4} \cdot \frac{x^{-2}}{x^{-2}} = \frac{x^{-2}+1}{x^{-2}+x^2} = \frac{1+x^{-2}}{x^2+x^{-2}} = \frac{1+x^{-2}}{(x-x^{-1})^2+2}$$

$$(x-x^{-1})^2 = x^2 - 2 + x^{-2} \Rightarrow (x-x^{-1})^2 + 2 = x^2 + x^{-2}$$

$$\frac{1+x^2}{1+x^4} dx = \frac{1+x^{-2}}{(x-x^{-1})^2+2} dx \rightarrow u = x - x^{-1} \\ du = (1+x^{-2}) dx = \frac{du}{u^2+2}$$

$$\int \frac{1+x^2}{1+x^4} dx = \int \frac{du}{u^2+2} = \frac{1}{2} \int \frac{du}{\left( \frac{u}{\sqrt{2}} \right)^2 + 1} \rightarrow t = \frac{u}{\sqrt{2}} = \frac{\sqrt{2}}{2} \int \frac{\frac{1}{\sqrt{2}} du}{\left( \frac{u}{\sqrt{2}} \right)^2 + 1} =$$

$$= \frac{\sqrt{2}}{2} \int \frac{dt}{t^2+1} = \frac{\sqrt{2}}{2} \operatorname{arctg}(t) + C = \frac{\sqrt{2}}{2} \operatorname{arctg} \left( \frac{u}{\sqrt{2}} \right) + C = \frac{\sqrt{2}}{2} \operatorname{arctg} \left( \frac{x-x^{-1}}{\sqrt{2}} \right) + C$$