מהו טור מקלורן של $f(x)=\frac{x}{x^2-5x+6}$?

ומהו תחום התכנסותו?

תשובות:

יש לבחור תשובה אחת:

a. $[-2,2),\ \sum_{n=1}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)x^n$

b. $(-2,2),\ \sum_{n=1}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)x^n$

c. $[-2,2],\ \sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{3^n}\right)x^n$

d. $(-2,2),\ \sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{3^n}\right)x^n$

$$\frac{x}{(x-3)(x-2)} = \frac{A}{x-3}+\frac{B}{x-2} =$$

$$= \frac{A(x-2)+B(x-3)}{(x-3)(x-2)} =$$

$$= \frac{(A+B)x-2A-3B}{(x-3)(x-2)}$$

$$\begin{cases} A+B=1 \\ -2A-3B=0 \end{cases} \Rightarrow\ B=-2\ ,\ A=3$$

$$\frac{x}{(x-3)(x-2)} = \frac{3}{x-3}-\frac{2}{x-2} =$$

$$= \frac{2}{2-x}-\frac{3}{3-x} = \frac{1}{1-x/2}-\frac{1}{1-x/3}$$

$$f_{(x)} = \frac{1}{1-x/2}-\frac{1}{1-x/3}$$

$$\frac{1}{1-x/2} = \sum\left(\frac{x}{2}\right)^{n-1} = 1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+\cdots+\left(\frac{x}{2}\right)^{n-1}+\cdots \qquad -2<x<2$$

$$\frac{1}{1-x/3} = \sum\left(\frac{x}{3}\right)^{n-1} = 1+\frac{x}{3}+\left(\frac{x}{3}\right)^2+\left(\frac{x}{3}\right)^3+\cdots+\left(\frac{x}{3}\right)^{n-1}+\cdots \qquad -3<x<3$$

$$f_{(x)} = \frac{1}{1-x/2}-\frac{1}{1-x/3} = \sum\left(\frac{x}{2}\right)^{n-1}-\sum\left(\frac{x}{3}\right)^{n-1} = \sum\left(\frac{1}{2^{n-1}}-\frac{1}{3^{n-1}}\right)x^{n-1} =$$

$$= \sum_{0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{3^n}\right)x^n \qquad , \qquad -2<x<2$$