נתון כי $u(x,y,z)=x^2 f\left(\frac{y}{x},\frac{z}{x}\right)$ נתון כי f דיפרנציאבלית בתחום הגדרתה.

מצא את $xu_x+yu_y+zu_z$.

תשובות:

יש לבחור תשובה אחת:

- a. $xu_x+yu_y+zu_z=xyzu$
- b. $xu_x+yu_y+zu_z=2u$
- c. $xu_x+yu_y+zu_z=2xu$
- d. $xu_x+yu_y+zu_z=u^2$

$$u_{(x,y,z)} = x^2 f_{\left(\frac{y}{x},\frac{z}{x}\right)}$$

$$t=\frac{y}{x} \Rightarrow \frac{\partial t}{\partial x}=\frac{-y}{x^2}\ ,\ \frac{\partial t}{\partial y}=\frac{1}{x}\ ,\ \frac{\partial t}{\partial z}=0$$

$$v=\frac{z}{x} \Rightarrow \frac{\partial v}{\partial x}=\frac{-z}{x^2}\ ,\ \frac{\partial v}{\partial y}=0\ ,\ \frac{\partial v}{\partial z}=\frac{1}{x}$$

$$\frac{\partial u}{\partial x} = 2x\cdot f_{(t,v)} + x^2\cdot\frac{\partial}{\partial x}f_{(t,v)}$$

$$\frac{\partial}{\partial x}f_{(t,v)} = \frac{\partial f_{(t,v)}}{\partial t}\cdot\frac{\partial t}{\partial x}+\frac{\partial f_{(t,v)}}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$\frac{\partial}{\partial x}f_{(t,v)} = \frac{\partial f_{(t,v)}}{\partial t}\cdot\frac{-y}{x^2}+\frac{\partial f_{(t,v)}}{\partial v}\cdot\frac{-z}{x^2} = \frac{-1}{x^2}\left(y\cdot\frac{\partial f_{(t,v)}}{\partial t}+z\cdot\frac{\partial f_{(t,v)}}{\partial v}\right)$$

$$\frac{\partial u}{\partial x} = 2x\cdot f_{(t,v)} + x^2\cdot\frac{-1}{x^2}\left(y\cdot\frac{\partial f_{(t,v)}}{\partial t}+z\cdot\frac{\partial f_{(t,v)}}{\partial v}\right) = 2x\cdot f_{(t,v)} - y\cdot\frac{\partial f_{(t,v)}}{\partial t} - z\cdot\frac{\partial f_{(t,v)}}{\partial v}$$

$$\frac{\partial u}{\partial y} = x^2\cdot\frac{\partial}{\partial y}f_{(t,v)}$$

$$\frac{\partial}{\partial y}f_{(t,v)} = \frac{\partial f_{(t,v)}}{\partial t}\cdot\frac{\partial t}{\partial y}+\frac{\partial f_{(t,v)}}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{\partial f_{(t,v)}}{\partial t}\cdot\frac{1}{x} = \frac{1}{x}\cdot\frac{\partial f_{(t,v)}}{\partial t}$$

$$\frac{\partial u}{\partial y} = x^2\cdot\frac{\partial}{\partial y}f_{(t,v)} = x^2\cdot\frac{1}{x}\cdot\frac{\partial f_{(t,v)}}{\partial t} = x\cdot\frac{\partial f_{(t,v)}}{\partial t}$$

$$\frac{\partial u}{\partial z} = x^2 \cdot \frac{\partial}{\partial z} f_{(t,v)}$$

$$\frac{\partial}{\partial z} f_{(t,v)} = \frac{\partial f_{(t,v)}}{\partial t} \cdot \frac{\partial t}{\partial z} + \frac{\partial f_{(t,v)}}{\partial v} \cdot \frac{\partial v}{\partial z} = \frac{\partial f_{(t,v)}}{\partial v} \cdot \frac{1}{x} = \frac{1}{x} \cdot \frac{\partial f_{(t,v)}}{\partial v}$$

$$\frac{\partial u}{\partial z} = x^2 \cdot \frac{\partial}{\partial z} f_{(t,v)} = x^2 \cdot \frac{1}{x} \cdot \frac{\partial f_{(t,v)}}{\partial v} = x \cdot \frac{\partial f_{(t,v)}}{\partial v}$$

$$xu_x + yu_y + zu_z =$$

$$= x\left(2x \cdot f_{(t,v)} - y \cdot \frac{\partial f_{(t,v)}}{\partial t} - z \cdot \frac{\partial f_{(t,v)}}{\partial v}\right) + yx \cdot \frac{\partial f_{(t,v)}}{\partial t} + zx \cdot \frac{\partial f_{(t,v)}}{\partial v} =$$

$$= 2x^2 \cdot f_{(t,v)} - yx \cdot \frac{\partial f_{(t,v)}}{\partial t} - zx \cdot \frac{\partial f_{(t,v)}}{\partial v} + yx \cdot \frac{\partial f_{(t,v)}}{\partial t} + zx \cdot \frac{\partial f_{(t,v)}}{\partial v} = 2x^2 \cdot f_{(t,v)}$$

$$u_{(x,y,z)} = x^2 f_{\left(\frac{y}{x}, \frac{z}{x}\right)} \quad it's\ given$$

$$xu_x + yu_y + zu_z = 2x^2 \cdot f_{(t,v)} = 2u_{(x,y,z)}$$