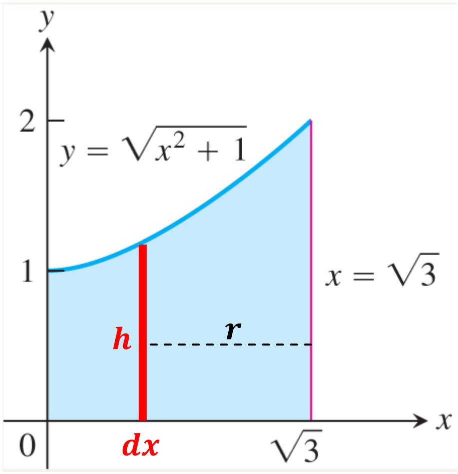


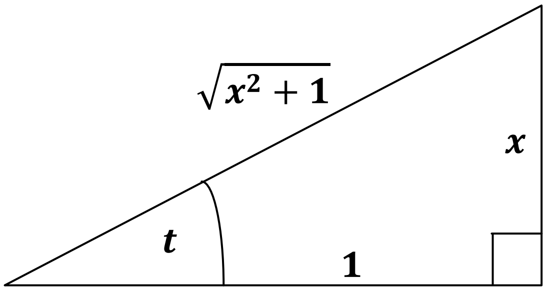
בשיטת הקליפות, הנפח המתקבל מסיבוב השטח סביב $x=\sqrt{3}$.

$$r_{(x)}=\sqrt{3}-x \quad , \quad h_{(x)}=y$$

$$dV = 2\pi r\cdot h\cdot dx = 2\pi(\sqrt{3}-x)\cdot\sqrt{x^2+1}\cdot dx$$

$$V=\int_a^b dv = 2\pi\int_0^{\sqrt{3}}(\sqrt{3}-x)\cdot\sqrt{x^2+1}\cdot dx$$

$$V = 2\pi\left[\sqrt{3}\int_0^{\sqrt{3}}\sqrt{x^2+1}\cdot dx - \int_0^{\sqrt{3}}x\sqrt{x^2+1}\cdot dx\right]$$



$$\cos t=\frac{1}{\sqrt{x^2+1}} \quad => \quad \sqrt{x^2+1}=\frac{1}{\cos t}$$

$$\tan t = x \quad => \quad dx=\frac{1}{\cos^2 t}\cdot dt$$

$$\int\sqrt{x^2+1}\,dx = \int\frac{1}{\cos t}\cdot\frac{1}{\cos^2 t}\cdot dt = \int\frac{1}{\cos^3 t}\cdot dt$$

$$\begin{cases} u=\dfrac{1}{\cos t} \quad => \quad du=\dfrac{\sin t}{\cos^2 t}dt \\ dv=\dfrac{1}{\cos^2 t}\cdot dt \quad => \quad v=\displaystyle\int\frac{1}{\cos^2 t}\cdot dt=\tan t \end{cases}$$

$$\int u\cdot dv = u\cdot v-\int v\cdot du$$

$$\int\frac{1}{\cos^3 t}\cdot dt=\frac{1}{\cos t}\cdot\tan t-\int\tan t\cdot\frac{\sin t}{\cos^2 t}dt$$

$$\int\tan t\cdot\frac{\sin t}{\cos^2 t}dt = \int\frac{\sin^2 t}{\cos^3 t}dt = \int\frac{1-\cos^2 t}{\cos^3 t}dt = \int\frac{1}{\cos^3 t}dt-\int\frac{1}{\cos t}dt$$

$$\int\frac{1}{\cos^3 t}\cdot dt = \frac{1}{\cos t}\cdot\tan t - \int\frac{1}{\cos^3 t}dt + \int\frac{1}{\cos t}dt$$

$$2\int \frac{1}{\cos^3 t}\cdot dt = \frac{1}{\cos t}\cdot \tan t + \int \frac{1}{\cos t}dt$$

$$\int \frac{1}{\cos t}dt = \int \frac{\cos t}{\cos^2 t}dt = \int \frac{\cos t}{1-\sin^2 t}dt \;\; -> \begin{cases} u = \sin t \\ du = \cos t \cdot dt \end{cases} -> \;\; = \int \frac{1}{1-u^2}du$$

$$\frac{1}{1-u^2} = \frac{1}{(1-u)(1+u)} = \frac{A}{1-u} + \frac{B}{1+u} = \frac{A(1+u)+B(1-u)}{(1-u)(1+u)} = \frac{(A-B)u + A + B}{(1+u)(1-u)}$$

$$\frac{1}{(1-u)(1+u)} = \frac{(A-B)u+A+B}{(1+u)(1-u)} \;\; => \;\; \begin{cases} A - B = 0 \\ A + B = 1 \end{cases} \;\; => \;\; A = B = \frac{1}{2}$$

$$\frac{1}{1-u^2} = \frac{A}{1-u} + \frac{B}{1+u} = \frac{1/2}{1-u} + \frac{1/2}{1+u} = \frac{1}{2}\left(\frac{1}{1-u} + \frac{1}{1+u}\right) = \frac{1}{2}\left(\frac{1}{u+1} - \frac{1}{u-1}\right)$$

$$\int \frac{1}{1-u^2}du = \frac{1}{2}\int \left(\frac{1}{u+1} - \frac{1}{u-1}\right)du = \frac{1}{2}(\ln|u+1| - \ln|u-1|) = \frac{1}{2}\ln\left|\frac{u+1}{u-1}\right|$$

$$2\int \frac{1}{\cos^3 t}\cdot dt = \frac{1}{\cos t}\cdot \tan t + \frac{1}{2}\ln\left|\frac{\sin t + 1}{\sin t - 1}\right|$$

$$\int \frac{1}{\cos^3 t}\cdot dt = \frac{\tan t}{2\cos t} + \frac{1}{4}\ln\left|\frac{\sin t + 1}{\sin t - 1}\right|$$

$$\int_0^{\sqrt{3}} \sqrt{x^2+1}\cdot dx = \int_0^{\pi/3} \frac{1}{\cos^3 t}\cdot dt = \left[\frac{\tan t}{2\cos t} + \frac{1}{4}\ln\left|\frac{\sin t + 1}{\sin t - 1}\right|\right|_0^{\pi/3} =$$

$$= \frac{\tan \pi/3}{2\cos \pi/3} + \frac{1}{4}\ln\left|\frac{\sin \pi/3 + 1}{\sin \pi/3 - 1}\right| - \left(\frac{\tan 0}{2\cos 0} + \frac{1}{4}\ln\left|\frac{\sin 0 + 1}{\sin 0 - 1}\right|\right) =$$

$$= \frac{\sqrt{3}}{2\cdot 1/2} + \frac{1}{4}\ln\left|\frac{\sqrt{3}/2 + 1}{\sqrt{3}/2 - 1}\right| - 0 = \sqrt{3} + \frac{1}{4}\ln\left|\frac{\sqrt{3}+2}{\sqrt{3}-2}\right| \approx 2.39$$

כעת לחישוב האינטגרל השני, הפשוט בהרבה:

$$\int_0^{\sqrt{3}} x\sqrt{x^2+1}\cdot dx = \frac{1}{2}\int_0^{\sqrt{3}} \sqrt{x^2+1}\cdot 2x\cdot dx \quad -> \quad \begin{cases} u = x^2+1 \\ du = 2x\cdot dx \end{cases} \quad -> \quad = \frac{1}{2}\int_1^4 \sqrt{u}\cdot du =$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_1^4 = \frac{1}{3}\left(4^{3/2}-1\right) = \frac{1}{3}(8-1) = \frac{7}{3}$$

לסיכום:

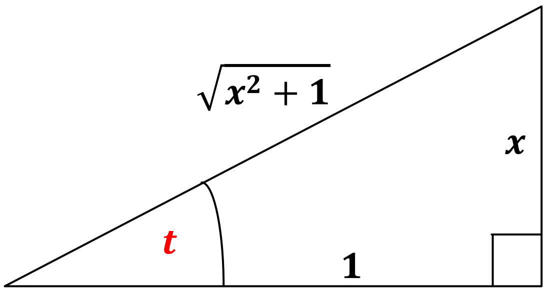
$$V = 2\pi\left[\sqrt{3}\int_0^{\sqrt{3}} \sqrt{x^2+1}\cdot dx - \int_0^{\sqrt{3}} x\sqrt{x^2+1}\cdot dx\right] =$$

$$= 2\pi\left[\sqrt{3}\cdot 2.39 - \frac{7}{3}\right] = 11.349\ \ Cubic\ Units$$

את המקום שנותר ננצל כדי לרשום (שוב) את הפונקציה הקדומה של $\frac{1}{cos^3 t}$:

$$F_{(t)} = \int \frac{1}{cos^3 t}\cdot dt = \frac{tan\ t}{2cos\ t} + \frac{1}{4}ln\left|\frac{sin\ t+1}{sin\ t-1}\right| + C$$

וממנה נמשיך כדי לרשום את הפונקציה הקדומה של $\sqrt{x^2+1}$:



$$tan\ t = x$$

$$sin\ t = \frac{x}{\sqrt{x^2+1}}$$

$$cos\ t = \frac{1}{\sqrt{x^2+1}}$$

$$F_{(x)} = \int \sqrt{x^2+1}\cdot dt = \frac{x}{2\frac{1}{\sqrt{x^2+1}}} + \frac{1}{4}ln\left|\frac{\frac{x}{\sqrt{x^2+1}}+1}{\frac{x}{\sqrt{x^2+1}}-1}\right| + C =$$

$$= \frac{x\sqrt{x^2+1}}{2} + \frac{1}{4}ln\left|\frac{x+\sqrt{x^2+1}}{x-\sqrt{x^2+1}}\right| + C$$