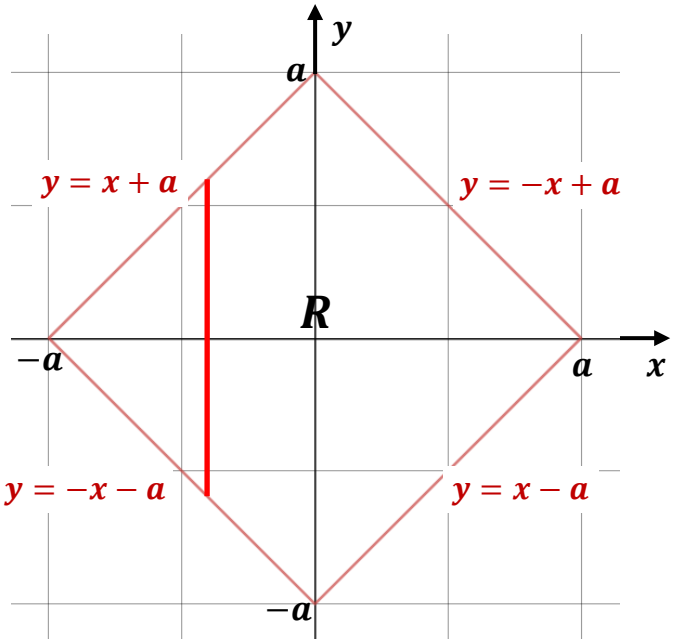


חשב את $\iint_{|x|+|y|\le a} e^{x+y}\,dA$

$$\iint_{|x|+|y|\le a} e^{x+y}dA = \int_{-a}^{0}\int_{-x-a}^{x+a} e^{x+y}\,dydx + \int_{0}^{a}\int_{x-a}^{-x+a} e^{x+y}\,dydx$$

$$\int_{-a}^{0}\int_{-x-a}^{x+a} e^{x+y}\,dydx = \int_{-a}^{0} e^{x+y}\Big|_{-x-a}^{x+a}dx = \int_{-a}^{0}(e^{x+x+a}-e^{x-x-a})\,dx =$$

$$= \int_{-a}^{0}(e^{2x+a}-e^{-a})\,dx = \left(\frac{e^{2x+a}}{2}-xe^{-a}\right)\Big|_{-a}^{0} = \frac{e^{a}}{2}-0-\left(\frac{e^{-a}}{2}+ae^{-a}\right) =$$

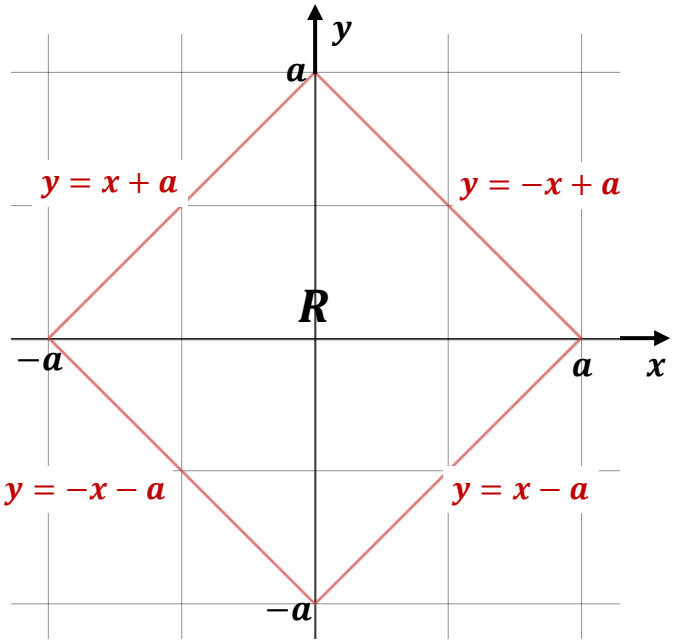
$$= \frac{e^{a}-e^{-a}}{2}-ae^{-a}$$

$$\int_{0}^{a}\int_{x-a}^{-x+a} e^{x+y}\,dydx = \int_{0}^{a} e^{x+y}\Big|_{x-a}^{-x+a}dx = \int_{0}^{a}(e^{x-x+a}-e^{x+x-a})\,dx =$$

$$= \int_{0}^{a}(e^{a}-e^{2x-a})\,dx = \left(xe^{a}-\frac{e^{2x-a}}{2}\right)\Big|_{0}^{a} = ae^{a}-\frac{e^{a}}{2}-\left(0-\frac{e^{-a}}{2}\right) =$$

$$= ae^{a}-\frac{e^{a}-e^{-a}}{2}$$

$$\iint_{|x|+|y|\le a} e^{x+y}dA = \frac{e^{a}-e^{-a}}{2}-ae^{-a}+ae^{a}-\frac{e^{a}-e^{-a}}{2} = ae^{a}-ae^{-a} = a(e^{a}-e^{-a})$$

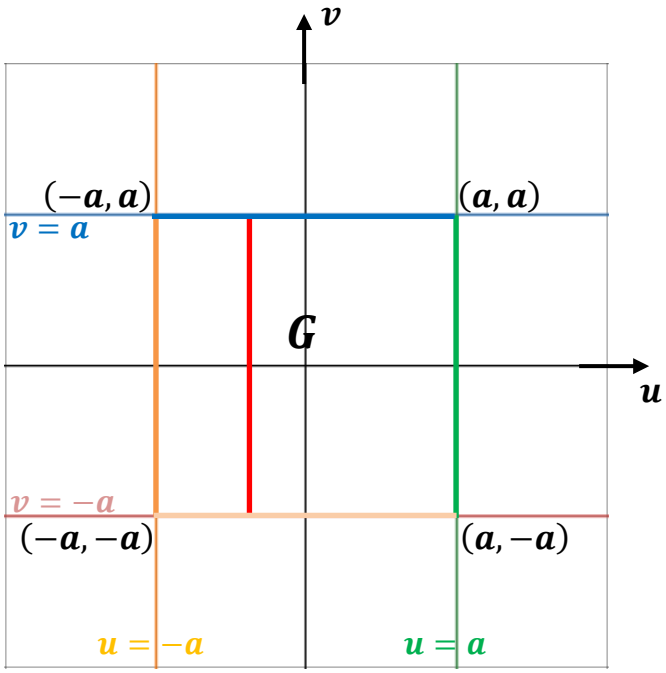


חשב את $\iint_{|x|+|y|\le a} e^{x+y}\,dA$

$$\begin{cases} u = y - x \\ v = y + x \end{cases} \Rightarrow \quad y = \frac{u+v}{2} \;,\; x = \frac{v-u}{2}$$

$$J_{(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$$

משוואות uv מפושטות	משוואות uv עבור הגבולות של G	משוואות xy עבור הגבולות של R
$u = a$	$\frac{u+v}{2} = \frac{v-u}{2} + a$	$y = x + a$
$v = -a$	$\frac{u+v}{2} = \frac{u-v}{2} - a$	$y = -x - a$
$v = a$	$\frac{u+v}{2} = \frac{u-v}{2} + a$	$y = -x + a$
$u = -a$	$\frac{u+v}{2} = \frac{v-u}{2} - a$	$y = x - a$



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$$\iint_{|x|+|y|\le a} e^{x+y}dA = \frac{1}{2}\int_{-a}^{a}\int_{-a}^{a} e^{v}\,dvdu =$$

$$= \frac{1}{2}\int_{-a}^{a} e^{v}\Big|_{-a}^{a} du = \frac{1}{2}\int_{-a}^{a}(e^{a} - e^{-a})\,du = \frac{1}{2}(e^{a} - e^{-a})u\Big|_{-a}^{a} = \frac{1}{2}(e^{a} - e^{-a})(a + a) = a(e^{a} - e^{-a})$$