נתון כי $w=f(x,y)$ וכן נתון כי בקורדינטות פולריות: $x=r\cos\theta$ ו $y=r\sin\theta$

מצא את $f_x^2+f_y^2$

c. $f_x^2+f_y^2=\left(\frac{\vartheta w}{\vartheta r}\right)^2+\frac{1}{r^2}\left(\frac{\vartheta w}{\vartheta \theta}\right)^2$

d. $f_x^2+f_y^2=\left(\frac{\vartheta w}{\vartheta \theta}\right)^2+\frac{1}{r^2}\left(\frac{\vartheta w}{\vartheta r}\right)^2$

$$\frac{\partial f}{\partial r}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r}=\frac{\partial f}{\partial x}\cdot\cos\theta+\frac{\partial f}{\partial y}\cdot\sin\theta$$

$$\frac{\partial f}{\partial \theta}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta}=\frac{\partial f}{\partial x}\cdot(-r\sin\theta)+\frac{\partial f}{\partial y}\cdot r\cos\theta$$

$$\left(\frac{\partial f}{\partial r}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2\cos^2\theta+\left(\frac{\partial f}{\partial y}\right)^2\sin^2\theta+2\frac{\partial f}{\partial x}\cdot\cos\theta\cdot\frac{\partial f}{\partial y}\cdot\sin\theta$$

$$\left(\frac{\partial f}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2 r^2\sin^2\theta+\left(\frac{\partial f}{\partial y}\right)^2 r^2\cos^2\theta-2r^2\frac{\partial f}{\partial x}\cdot\sin\theta\cdot\frac{\partial f}{\partial y}\cdot\cos\theta$$

$$\left(\frac{\partial f}{\partial r}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2\cos^2\theta+\left(\frac{\partial f}{\partial y}\right)^2\sin^2\theta+\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\cdot\sin 2\theta$$

$$\left(\frac{\partial f}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2 r^2\sin^2\theta+\left(\frac{\partial f}{\partial y}\right)^2 r^2\cos^2\theta-r^2\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\cdot\sin 2\theta$$

$$\left(\frac{\partial f}{\partial r}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2\cos^2\theta+\left(\frac{\partial f}{\partial y}\right)^2\sin^2\theta+\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\cdot\sin 2\theta$$

$$\frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2\sin^2\theta+\left(\frac{\partial f}{\partial y}\right)^2\cos^2\theta-\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\cdot\sin 2\theta$$

חיבור המשוואות מניב:

$$\left(\frac{\partial f}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2(\cos^2\theta+\sin^2\theta)+\left(\frac{\partial f}{\partial y}\right)^2(\sin^2\theta+\cos^2\theta)$$

$$\left(\frac{\partial f}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2=\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2$$

$$\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2=f_x^2+f_y^2$$