יהיו $\varphi(t)$ ו $\psi(t)$ גזירות פעמיים.

נגדיר: $z(x,y)=x\varphi(x+y)+y\psi(x+y)$

מצא את ערכו של $\frac{\vartheta^2 z}{\vartheta x \vartheta y}$ בנקודה כלשהי.

תשובות:

יש לבחור תשובה אחת:

a. $\frac{\vartheta^2 z}{\vartheta x \vartheta y}=x\cdot\frac{\vartheta\varphi}{\vartheta t}+\frac{\vartheta^2\varphi}{\vartheta t^2}+y\frac{\vartheta\psi}{\vartheta t}+\frac{\vartheta^2\psi}{\vartheta t^2}$

b. $\frac{\vartheta^2 z}{\vartheta x \vartheta y}=\frac{\vartheta\varphi}{\vartheta t}-y\cdot\frac{\vartheta^2\varphi}{\vartheta t^2}+\frac{\vartheta\psi}{\vartheta t}+x\frac{\vartheta^2\psi}{\vartheta t^2}$

c. $\frac{\vartheta^2 z}{\vartheta x \vartheta y}=\frac{\vartheta\varphi}{\vartheta t}+y\cdot\frac{\vartheta^2\varphi}{\vartheta t^2}+\frac{\vartheta\psi}{\vartheta t}+x\frac{\vartheta^2\psi}{\vartheta t^2}$

d. $\frac{\vartheta^2 z}{\vartheta x \vartheta y}=\frac{\vartheta\varphi}{\vartheta t}+x\frac{\vartheta^2\varphi}{\vartheta t^2}+\frac{\vartheta\psi}{\vartheta t}+y\frac{\vartheta^2\psi}{\vartheta t^2}$

$$z_{(x,y)} = x\varphi_{(x+y)} + y\psi_{(x+y)} \quad \leftarrow \quad t = x+y$$

$$z_{(x,y)} = x\varphi_{(t)} + y\psi_{(t)} \qquad \frac{\partial t}{\partial y} = \frac{\partial t}{\partial x} = 1$$

$$\frac{\partial z}{\partial y} = x\cdot\frac{\partial\varphi_{(t)}}{\partial y} + \left(1\cdot\psi_{(t)} + y\cdot\frac{\partial\psi_{(t)}}{\partial y}\right)$$

לתשומת לבכם :

$$\varphi_{(t_{(y)})} \quad => \quad \frac{\partial\varphi_{(t)}}{\partial y} = \frac{\partial\varphi_{(t)}}{\partial t}\cdot\frac{\partial t}{\partial y} = \frac{\partial\varphi_{(t)}}{\partial t}$$

$$\psi_{(t_{(y)})} \quad => \quad \frac{\partial\psi_{(t)}}{\partial y} = \frac{\partial\psi_{(t)}}{\partial t}\cdot\frac{\partial t}{\partial y} = \frac{\partial\psi_{(t)}}{\partial t}$$

ולכן אפשר לרשום את אגף ימין כדלקמן :

$$\frac{\partial z}{\partial y} = x\cdot\frac{\partial\varphi_{(t)}}{\partial t} + \psi_{(t)} + y\cdot\frac{\partial\psi_{(t)}}{\partial t}$$

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial}{\partial x}\left(x\cdot\frac{\partial\varphi_{(t)}}{\partial t}\right) + \frac{\partial\psi_{(t)}}{\partial x} + \frac{\partial}{\partial x}\left(y\cdot\frac{\partial\psi_{(t)}}{\partial t}\right)$$

$$\frac{\partial^2 z}{\partial x\partial y} = \left(1\cdot\frac{\partial\varphi_{(t)}}{\partial t} + x\cdot\frac{\partial}{\partial x}\frac{\partial\varphi_{(t)}}{\partial t}\right) + \frac{\partial\psi_{(t)}}{\partial t} + y\cdot\frac{\partial}{\partial x}\frac{\partial\psi_{(t)}}{\partial t}$$

$$\frac{\partial^2 z}{\partial x\partial y} = \left(\varphi_t + x\cdot\frac{\partial}{\partial x}\varphi_t\right) + \psi_t + y\cdot\frac{\partial}{\partial x}\psi_t \quad \leftarrow \quad \textit{Note! do not confuse } f_{(t)} \textit{ with } f_t$$

$$\frac{\partial^2 z}{\partial x\partial y} = \varphi_t + x\cdot\frac{\partial\varphi_t}{\partial x} + \psi_t + y\cdot\frac{\partial\psi_t}{\partial x} \quad \leftarrow \quad \textit{more compactly written}$$

שוב, לתשומת לבכם :

$$\varphi_{t(t_{(x)})} \quad => \quad \frac{\partial\varphi_t}{\partial x} = \frac{\partial\varphi_t}{\partial t}\cdot\frac{\partial t}{\partial x} = \frac{\partial\varphi_t}{\partial t}$$

$$\psi_{t(t_{(x)})} \quad => \quad \frac{\partial\psi_t}{\partial x} = \frac{\partial\psi_t}{\partial t}\cdot\frac{\partial t}{\partial x} = \frac{\partial\psi_t}{\partial t}$$

ולכן אפשר לרשום את אגף ימין כך :

$$\frac{\partial^2 z}{\partial x\partial y} = \varphi_t + x\cdot\frac{\partial\varphi_t}{\partial t} + \psi_t + y\cdot\frac{\partial\psi_t}{\partial t}$$

או בכתיב אחר: $$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial\varphi}{\partial t} + x\cdot\frac{\partial^2\varphi}{\partial t^2} + \frac{\partial\psi}{\partial t} + y\cdot\frac{\partial^2\psi}{\partial t^2}$$