$$z = f_{(x\cdot y)} + g_{\left(\frac{x}{y}\right)}$$

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} \qquad \leftarrow \qquad \textit{simplify the expression}$$

$$z = f_{(t)} + g_{(v)} \qquad t = x\cdot y \;=>\; \frac{\partial t}{\partial x} = y \;,\; \frac{\partial t}{\partial y} = x$$

$$v = \frac{y}{x} \;=>\; \frac{\partial v}{\partial x} = \frac{-y}{x^2} \;,\; \frac{\partial v}{\partial y} = \frac{1}{x}$$

$$\frac{\partial z}{\partial x} = \frac{\partial f_{(t)}}{\partial t}\cdot\frac{\partial t}{\partial x} + \frac{\partial g_{(v)}}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f_{(t)}}{\partial t}\cdot y + \frac{\partial g_{(v)}}{\partial v}\cdot\frac{-y}{x^2} = y\left(\frac{\partial f_{(t)}}{\partial t} - \frac{1}{x^2}\cdot\frac{\partial g_{(v)}}{\partial v}\right)$$

$$\frac{\partial z}{\partial x} = y\left(f_t - \frac{1}{x^2}\cdot g_v\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left[y\left(f_t - \frac{1}{x^2}\cdot g_v\right)\right] = y\frac{\partial}{\partial x}\left(f_t - \frac{1}{x^2}\cdot g_v\right) =$$

$$= y\left[\frac{\partial}{\partial x}(f_t) - \frac{\partial}{\partial x}\left(\frac{1}{x^2}\cdot g_v\right)\right] = y\left[\frac{\partial f_t}{\partial t}\cdot\frac{\partial t}{\partial x} - \left(\frac{-2}{x^3}\cdot g_v + \frac{1}{x^2}\cdot\frac{\partial g_v}{\partial v}\cdot\frac{\partial v}{\partial x}\right)\right] =$$

$$= y\left[\frac{\partial f_t}{\partial t}\cdot y - \left(\frac{-2}{x^3}\cdot g_v + \frac{1}{x^2}\cdot\frac{\partial g_v}{\partial v}\cdot\frac{-y}{x^2}\right)\right] = y\left[y\cdot\frac{\partial f_t}{\partial t} + \frac{2}{x^3}\cdot g_v + \frac{y}{x^4}\cdot\frac{\partial g_v}{\partial v}\right]$$

$$\frac{\partial^2 z}{\partial x^2} = y^2\cdot f_{tt} + \frac{2y}{x^3}\cdot g_v + \frac{y^2}{x^4}\cdot g_{vv}$$

$$\frac{\partial z}{\partial y} = \frac{\partial f_{(t)}}{\partial t}\cdot\frac{\partial t}{\partial y} + \frac{\partial g_{(v)}}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{\partial f_{(t)}}{\partial t}\cdot x + \frac{\partial g_{(v)}}{\partial v}\cdot\frac{1}{x} = x\cdot f_t + \frac{1}{x}\cdot g_v$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left[x\cdot f_t + \frac{1}{x}\cdot g_v\right] = x\cdot\frac{\partial f_t}{\partial y} + \frac{1}{x}\cdot\frac{\partial g_v}{\partial y} = x\cdot\frac{\partial f_t}{\partial t}\cdot\frac{\partial t}{\partial y} + \frac{1}{x}\cdot\frac{\partial g_v}{\partial v}\cdot\frac{\partial v}{\partial y} =$$

$$= x\cdot\frac{\partial f_t}{\partial t}\cdot x + \frac{1}{x}\cdot\frac{\partial g_v}{\partial v}\cdot\frac{1}{x} \quad => \quad \frac{\partial^2 z}{\partial y^2} = x^2\cdot f_{tt} + \frac{1}{x^2}\cdot g_{vv}$$

$$x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y} =$$

$$= x^2\left(y^2\cdot f_{tt} + \frac{2y}{x^3}\cdot g_v + \frac{y^2}{x^4}\cdot g_{vv}\right) - y^2\left(x^2\cdot f_{tt} + \frac{1}{x^2}\cdot g_{vv}\right) +$$

$$+ xy\left(f_t - \frac{1}{x^2}\cdot g_v\right) - y\left(x\cdot f_t + \frac{1}{x}\cdot g_v\right) =$$

$$= x^2y^2\cdot f_{tt} + \frac{2y}{x}\cdot g_v + \frac{y^2}{x^2}\cdot g_{vv} - y^2x^2\cdot f_{tt} - \frac{y^2}{x^2}\cdot g_{vv} +$$

$$+ xyf_t - \frac{y}{x}\cdot g_v - yx\cdot f_t - \frac{y}{x}\cdot g_v =$$

$$= x^2y^2\cdot f_{tt} - y^2x^2\cdot f_{tt} + \frac{2y}{x}\cdot g_v - \frac{y}{x}\cdot g_v - \frac{y}{x}\cdot g_v + \frac{y^2}{x^2}\cdot g_{vv} - \frac{y^2}{x^2}\cdot g_{vv} +$$

$$+ xyf_t - yx\cdot f_t = 0$$