מצא את $\lim_{x\to 0}\frac{\sin x-arctg x}{x^2\cdot\ln(1+x)}$ תוך פיתוח לטור חזקות של הביטויים הרצויים.

$$\sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}\cdot x^k = f(0)+f'(0)\cdot x+\frac{f''(0)}{2!}\cdot x^2+\cdots+\frac{f^{(n)}(0)}{n!}\cdot x^n+\cdots$$

$$f_{(x)} = \sin(x) \;\;=>\;\; f_{(0)} = 0 \;\;,\;\; f'_{(x)} = \cos(x) \;\;=>\;\; f'_{(0)} = 1$$

$$f''_{(x)} = -\sin(x) \;\;=>\;\; f''_{(0)} = 0$$

$$f^{3}_{(x)} = -\cos(x) \;\;=>\;\; f''_{(0)} = -1$$

$$\sin(x) \approx x-\frac{1}{3!}\cdot x^3 \quad \textit{for small values of } x$$

$$f_{(x)} = arctg(x) \;\;=>\;\; f_{(0)} = 0 \;\;,\;\; f'_{(x)} = \frac{1}{1+x^2} \;\;=>\;\; f'_{(0)} = 1$$

$$f''_{(x)} = -2\frac{x}{(1+x^2)^2} \;\;=>\;\; f''_{(0)} = 0$$

$$f^{(3)}_{(x)} = -2\frac{(1+x^2)^2-4x^2(1+x^2)}{(1+x^2)^4} = -2\frac{1+x^2-4x^2}{(1+x^2)^3} = -2\frac{1-3x^2}{(1+x^2)^3} \;\;=>\;\; f^{(3)}_{(0)} = -2$$

$$arctg(x) \approx x-\frac{2}{3!}\cdot x^3 \quad \textit{for small values of } x$$

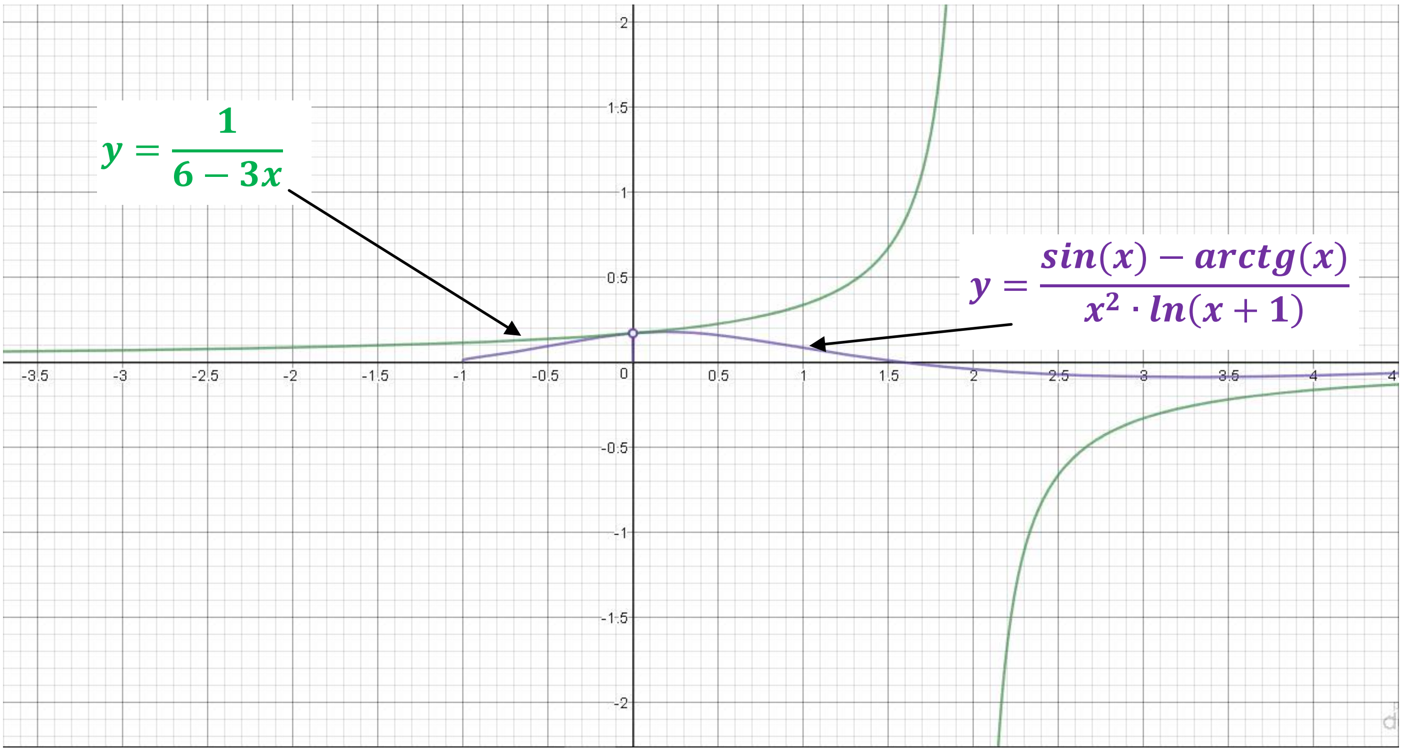
$$f_{(x)} = \ln(x+1) \;\;=>\;\; f_{(0)} = 0 \;\;,\;\; f'_{(x)} = \frac{1}{x+1} \;\;=>\;\; f'_{(0)} = 1$$

$$f''_{(x)} = -\frac{1}{(x+1)^2} \;\;=>\;\; f''_{(0)} = -1$$

$$\ln(x+1) \approx x-\frac{1}{2!}\cdot x^2 \quad \textit{for small values of } x$$

$$\lim_{x\to 0}\frac{\sin(x)-arctg(x)}{x^2\cdot\ln(x+1)} = \lim_{x\to 0}\frac{x-\frac{1}{3!}\cdot x^3-\left(x-\frac{2}{3!}\cdot x^3\right)}{x^2\cdot\left(x-\frac{1}{2!}\cdot x^2\right)} =$$

$$= \lim_{x\to 0}\frac{x^3}{x^2\cdot(6x-3x^2)} = \lim_{x\to 0}\frac{1}{6-3x} = \frac{1}{6}$$



$y = \frac{1}{6-3x}$ הוא קירוב טוב של $y = \frac{sin(x)-arctg(x)}{x^2 \cdot ln(x+1)}$ עבור ערכים קטנים של x, ז"א בקרבת ציר ה- y.