מצא את $\sum_{n=1}^{\infty}\left(\frac{1}{4n^2-1}\right)$

תשובה:

$$\sum_{n=1}^{\infty}\frac{1}{4n^2-1}=\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}+\cdots$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n-1)}=\frac{1}{3\cdot 1}+\frac{1}{5\cdot 3}+\frac{1}{7\cdot 5}+\frac{1}{9\cdot 7}+\frac{1}{11\cdot 9}+\cdots$$

$$\frac{1}{(2n+1)(2n-1)}=\frac{A}{2n+1}+\frac{B}{2n-1}=\frac{A(2n-1)+B(2n+1)}{(2n+1)(2n-1)}$$

$$1=A(2n-1)+B(2n+1)\quad =>\quad 1=2(A+B)n-A+B\quad =>\quad \begin{cases}A+B=0\\-A+B=1\end{cases}\quad =>\quad B=\frac{1}{2},\ A=-\frac{1}{2}$$

$$\frac{1}{(2n+1)(2n-1)}=\frac{-\frac{1}{2}}{2n+1}+\frac{\frac{1}{2}}{2n-1}=\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$$

$$\sum_{n=1}^{k}\frac{1}{(2n+1)(2n-1)}=\frac{1}{2}\sum_{n=1}^{k}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\cdots-\frac{1}{2k+1}\right)$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n-1)}=\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)=\frac{1}{2}\lim_{k\to\infty}\sum_{n=1}^{k}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)=$$

$$\frac{1}{2}\lim_{k\to\infty}\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\cdots-\frac{1}{2k+1}\right)=\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\cdots-\frac{1}{\infty}\right)=\frac{1}{2}$$

מי הוא השבר (מונה ומכנה) המסתתר מאחורי המספר הרציונלי $0.7282828...$?

תשובה: /

$$0.7282828\ldots=7\cdot 10^{-1}+28\cdot 10^{-3}+28\cdot 10^{-5}+28\cdot 10^{-7}+\cdots+28\cdot 10^{-(2n-1)}+\cdots$$

$$0.7282828\ldots=7\cdot 10^{-1}+28\left(10^{-3}+10^{-5}+10^{-7}+\cdots+10^{-(2n+1)}+\cdots\right)$$

$$S_{n\to\infty}=\frac{a_1}{1-q}\quad \textit{appropriate for any geometric series}$$

$$\{10^{-3}+10^{-5}+10^{-7}+\cdots+10^{-(2n+1)}+\cdots\}=\frac{10^{-3}}{1-10^{-2}}=\frac{1}{990}$$

$$0.7282828\ldots=7\cdot 10^{-1}+28\cdot\frac{1}{990}=\frac{7}{10}+\frac{28}{990}=\frac{693}{990}+\frac{28}{990}=\frac{721}{990}$$