$$f_{(x)} = ln(2x+1) \quad => \quad \frac{df}{dx} = \lim_{\Delta x \to 0}\left[\frac{f_{(x+\Delta x)} - f_{(x)}}{\Delta x}\right] =$$

$$= \lim_{\Delta x \to 0}\left[\frac{ln[2(x+\Delta x)+1] - ln(2x+1)}{\Delta x}\right] = \lim_{\Delta x \to 0}\left[\frac{1}{\Delta x} ln\left[\frac{2(x+\Delta x)+1}{2x+1}\right]\right] =$$

$$= \lim_{\Delta x \to 0}\left[\frac{1}{\Delta x} ln\left(\frac{2x+1+2\Delta x}{2x+1}\right)\right] = \lim_{\Delta x \to 0}\left[\frac{1}{\Delta x} ln\left(1+\frac{2\Delta x}{2x+1}\right)\right] =$$

$$= \lim_{\Delta x \to 0}\left[ln\left(1+\frac{2\Delta x}{2x+1}\right)^{\frac{1}{\Delta x}}\right] =$$

$$= \lim_{\Delta x \to 0}\left\{ln\left[\left(1+\frac{2\Delta x}{2x+1}\right)^{\frac{2x+1}{2\Delta x}\cdot\frac{2\Delta x}{2x+1}}\right]^{\frac{1}{\Delta x}}\right\} = \lim_{\Delta x \to 0}\left\{ln\left[\left(1+\frac{2\Delta x}{2x+1}\right)^{\frac{2x+1}{2\Delta x}}\right]^{\frac{2\Delta x}{2x+1}\cdot\frac{1}{\Delta x}}\right\} =$$

$$= ln\left\{\lim_{\Delta x \to 0}\left[\left(1+\frac{2\Delta x}{2x+1}\right)^{\frac{2x+1}{2\Delta x}}\right]^{\frac{2}{2x+1}}\right\} = ln\, e^{2/(2x+1)} = \frac{2}{2x+1}\cdot ln\, e = \frac{2}{2x+1}$$