$$f_{(x)} = x \sin\left(\frac{1}{x}\right) \quad => \quad \frac{df}{dx} = \lim_{\Delta x \to 0}\left[\frac{f_{(x+\Delta x)} - f_{(x)}}{\Delta x}\right] = \lim_{\Delta x \to 0}\left[\frac{(x+\Delta x)\cdot \sin\left(\frac{1}{x+\Delta x}\right) - x \sin\left(\frac{1}{x}\right)}{\Delta x}\right] =$$

$$= \lim_{\Delta x \to 0}\left\{\frac{1}{\Delta x}\left\{x\left[\sin\left(\frac{1}{x+\Delta x}\right) - \sin\left(\frac{1}{x}\right)\right] + \Delta x \cdot \sin\left(\frac{1}{x+\Delta x}\right)\right\}\right\} = \; / \; \sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$= \lim_{\Delta x \to 0}\left\{\frac{1}{\Delta x}\left[x \cdot 2\sin\left(\frac{-\Delta x}{2x(x+\Delta x)}\right)\cos\left(\frac{2x+\Delta x}{2x(x+\Delta x)}\right) + \Delta x \cdot \sin\left(\frac{1}{x+\Delta x}\right)\right]\right\} =$$

$$= -\lim_{\Delta x \to 0}\left\{\frac{1}{\Delta x}\left[x \cdot 2\sin\left(\frac{\Delta x}{2x(x+\Delta x)}\right)\cos\left(\frac{2x+\Delta x}{2x(x+\Delta x)}\right) - \Delta x \cdot \sin\left(\frac{1}{x+\Delta x}\right)\right]\right\} =$$

$$= -\lim_{\Delta x \to 0}\left\{\frac{1}{\Delta x}\left[2x\frac{\Delta x}{2x(x+\Delta x)} \cdot \frac{\sin\left(\frac{\Delta x}{2x(x+\Delta x)}\right)}{\frac{\Delta x}{2x(x+\Delta x)}}\cos\left(\frac{2x+\Delta x}{2x(x+\Delta x)}\right) - \Delta x \cdot \sin\left(\frac{1}{x+\Delta x}\right)\right]\right\} =$$

$$= -\lim_{\Delta x \to 0}\left[\frac{1}{x+\Delta x} \cdot \frac{\sin\left(\frac{\Delta x}{2x(x+\Delta x)}\right)}{\frac{\Delta x}{2x(x+\Delta x)}}\cos\left(\frac{2x+\Delta x}{2x(x+\Delta x)}\right) - \sin\left(\frac{1}{x+\Delta x}\right)\right] =$$

$$= -\left[\frac{1}{x} \cdot 1 \cdot \cos\left(\frac{2x}{2x(x)}\right) - \sin\left(\frac{1}{x}\right)\right] = \sin\left(\frac{1}{x}\right) - \frac{1}{x} \cdot \cos\left(\frac{1}{x}\right)$$