$$f_{(x)} = x^3 \sin\frac{1}{x} \quad => \quad \frac{df}{dx} = \lim_{\Delta x\to 0}\left[\frac{f_{(x+\Delta x)} - f_{(x)}}{\Delta x}\right] = \lim_{\Delta x\to 0}\left[\frac{(x+\Delta x)^3 \sin\frac{1}{x+\Delta x} - x^3 \sin\frac{1}{x}}{\Delta x}\right]$$

$$\frac{(x+\Delta x)^3 \sin\frac{1}{x+\Delta x} - x^3 \sin\frac{1}{x}}{\Delta x} = \frac{\left(x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3\right)\sin\frac{1}{x+\Delta x} - x^3 \sin\frac{1}{x}}{\Delta x} =$$

$$= \frac{\left(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3\right)\sin\frac{1}{x+\Delta x} + x^3 \sin\frac{1}{x+\Delta x} - x^3 \sin\frac{1}{x}}{\Delta x} =$$

$$= \frac{\left(3x^2\Delta x + 3x\Delta x^2 + \Delta x^3\right)\sin\frac{1}{x+\Delta x} + x^3\left(\sin\frac{1}{x+\Delta x} - \sin\frac{1}{x}\right)}{\Delta x} =$$

$$= (3x^2 + 3x\Delta x + \Delta x^2)\sin\frac{1}{x+\Delta x} + x^3\frac{\sin\frac{1}{x+\Delta x} - \sin\frac{1}{x}}{\Delta x} = \quad / \ \sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$= \left(3x^2 + 3x\Delta x + \Delta x^2\right)\sin\frac{1}{x+\Delta x} + x^3\frac{2\cos\frac{2x+\Delta x}{2x(x+\Delta x)}\sin\frac{-\Delta x}{2x(x+\Delta x)}}{\Delta x} =$$

$$= \left(3x^2 + 3x\Delta x + \Delta x^2\right)\sin\frac{1}{x+\Delta x} - 2x^3\frac{\cos\frac{2x+\Delta x}{2x(x+\Delta x)}\sin\frac{\Delta x}{2x(x+\Delta x)}}{\Delta x} =$$

$$= \left(3x^2 + 3x\Delta x + \Delta x^2\right)\sin\frac{1}{x+\Delta x} - \frac{1}{2x(x+\Delta x)}2x^3\cos\frac{2x+\Delta x}{2x(x+\Delta x)}\cdot\frac{\sin\frac{\Delta x}{2x(x+\Delta x)}}{\frac{\Delta x}{2x(x+\Delta x)}} =$$

$$= \left(3x^2 + 3x\Delta x + \Delta x^2\right)\sin\frac{1}{x+\Delta x} - \frac{1}{x+\Delta x}x^2\cos\frac{2x+\Delta x}{2x(x+\Delta x)}\cdot\frac{\sin\frac{\Delta x}{2x(x+\Delta x)}}{\frac{\Delta x}{2x(x+\Delta x)}} = \frac{f_{(x+\Delta x)} - f_{(x)}}{\Delta x}$$

$$\lim_{\Delta x\to 0}\left(\frac{f_{(x+\Delta x)} - f_{(x)}}{\Delta x}\right) = \lim_{\Delta x\to 0}\left[(3x^2 + 3x\Delta x + \Delta x^2)\sin\frac{1}{x+\Delta x}\right] - \lim_{\Delta x\to 0}\left[\frac{1}{x+\Delta x}x^2\cos\frac{2x+\Delta x}{2x(x+\Delta x)}\cdot\frac{\sin\frac{\Delta x}{2x(x+\Delta x)}}{\frac{\Delta x}{2x(x+\Delta x)}}\right] =$$

$$= \left[3x^2\sin\frac{1}{x}\right] - \left[\frac{1}{x}x^2\cos\frac{2x}{2x(x)}\cdot 1\right] = 3x^2\sin\frac{1}{x} - x\cos\frac{1}{x}$$