9. Area The area A of a triangle with sides of lengths a and b enclosing an angle of measure θ is

$$A = \frac{1}{2} ab \sin\theta .$$

a. How is dA/dt related to $d\theta/dt$ if a and b are constant?

b. How is dA/dt related to $d\theta/dt$ and da/dt if only b is constant?

c. How is dA/dt related to $d\theta/dt$, da/dt, and db/dt if none of a, b, and θ are constant?

א. a ו- b קבועים:

$$A = \frac{1}{2} ab \sin\theta \quad => \quad \frac{dA}{dt} = \frac{1}{2} ab \cos\theta \cdot \frac{d\theta}{dt}$$

ב. רק b קבוע:

$$A = \frac{1}{2} ab \sin\theta \quad => \quad \frac{dA}{dt} = \frac{1}{2} b \left[\frac{da}{dt} \sin\theta + a\cos\theta \cdot \frac{d\theta}{dt}\right]$$

ג. אף אחד לא קבוע:

$$A = \frac{1}{2} ab \sin\theta \quad => \quad \frac{dA}{dt} = \frac{1}{2} \left[\frac{da}{dt} b \sin\theta + \frac{db}{dt} a \sin\theta + ab\cos\theta \cdot \frac{d\theta}{dt}\right]$$

גבולות :

$$\lim_{x\to\infty}\left(\frac{x+2}{x-3}\right)^{2x+3} = \lim_{x\to\infty}\left(\frac{x-3+5}{x-3}\right)^{2x+3} = \lim_{x\to\infty}\left(1+\frac{5}{x-3}\right)^{2x+3} = \lim_{x\to\infty}\left[\left(1+\frac{5}{x-3}\right)^{\frac{x-3}{5}\cdot\frac{5}{x-3}}\right]^{2x+3} =$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{5}{x-3}\right)^{\frac{x-3}{5}}\right]^{\cdot\frac{5}{x-3}(2x+3)} = \lim_{x\to\infty}\left[\left(1+\frac{5}{x-3}\right)^{\frac{x-3}{5}}\right]^{\cdot\frac{10x+15}{x-3}} = \lim_{x\to\infty}\left[\left(1+\frac{5}{x-3}\right)^{\frac{x-3}{5}}\right]^{\cdot\frac{10+\frac{15}{x}}{1-\frac{3}{x}}} = e^{10}$$

$$\lim_{x\to\infty}\left(2-\frac{1}{4x}\right)^{x} = \lim_{x\to\infty}\left[2\left(1+\frac{-1}{8x}\right)\right]^{x} = \lim_{x\to\infty} 2^x \cdot \lim_{x\to\infty}\left(1+\frac{-1}{8x}\right)^{x} = \lim_{x\to\infty} 2^x \cdot \lim_{x\to\infty}\left[\left(1+\frac{-1}{8x}\right)^{\frac{8x}{-1}\cdot\frac{-1}{8x}}\right]^{x} =$$

$$= \lim_{x\to\infty} 2^x \cdot \lim_{x\to\infty}\left[\left(1+\frac{-1}{8x}\right)^{\frac{8x}{-1}}\right]^{\frac{-1}{8x}\cdot x} = \lim_{x\to\infty} 2^x \cdot \lim_{x\to\infty}\left[\left(1+\frac{-1}{8x}\right)^{\frac{8x}{-1}}\right]^{-\frac{1}{8}} = \infty \cdot e^{-1/8} = \infty$$